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Discussion Paper Series

Economics

1

A New Economic Framework: A DSGE Model with Cryptocurrency*

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July 17, 2020

Abstract

We develop and estimate a DSGE model to evaluate the economic repercussions of cryptocurrency. We assume that cryptocurrency offers an alternative currency option to government currency, with endogenous supply and demand. We uncover a substitution effect between the real balances of government currency and cryptocurrency in response to technology, preferences and monetary policy shocks. We also observe a countercyclical reaction of real balances of cryptocurrency to these shocks. Cryptocurrency productivity shocks have negative effects on output, inflation and cryptocurrency exchange rate. Finally, output and inflation responses are stronger when cryptocurrency is introduced in the utility function in a non-separable way.

Keywords: DSGE Model, Government Currency, Cryptocurrency, Bayesian Estimation.
JEL classification: E40, E41, E51, E52.

*We would like to thank Paolo Gelain, John Sessions, Harald Uhlig and the conference participants at the 12*th* Conference on Computational and Financial Econometrics, the 27*th* Annual Symposium of the Society for Nonlinear Dynamics and Econometrics, the 25*th* International Conference Computing in Economics and Finance, the 15*th* Dynare Conference and the University of Sydney Macroeconomic Reading Group Workshop.

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1 Introduction

Cryptocurrency has recently gained considerable interest from investors, central banks and governments worldwide. There are numerous reasons for this intensified attention. For example, Japan and South Korea have recognised Bitcoin as a legal method of payment (Bloomberg, 2017a; Cointelegraph, 2017). Some central banks are exploring the possibility of using cryptocurrency (Bloomberg, 2017c). Moreover, a large number of companies and banks have created the Enterprise Ethereum Alliance[1] in order to customise Ethereum for industry players (Forbes, 2017). Finally, the Chicago Mercantile Exchange (CME) started the Bitcoin futures on 18^{th} December 2017 (Chicago Mercantile Exchange, 2017).[2]

In this paper, we develop and estimate a Dynamic Stochastic General Equilibrium (DSGE) model in order to evaluate the economic repercussions of cryptocurrency. Our model includes the demand and supply of cryptocurrency by extending and reformulating standard DSGE models with money (see, among others, Nelson, 2002, Christiano et al., 2005, Ireland, 2004) with the new sector of the economy related to cryptocurrency. Our analysis allows us to compare the responses of real money balances for government currency and cryptocurrency to several demand and supply shocks driving the economy. Moreover, we are able to evaluate the response of the main macroeconomic fundamentals to productivity shocks for the production of cryptocurrency.

In 2017 the value of cryptocurrencies experienced exponential growth and their market capitalization substantially increased. However, the volatility of cryptocurrencies has been very significant with regular daily swings of up to 30%. Figure 1 provides evidence of these characteristics by showing the

[1]Source: https://entethalliance.org/members/.

[2]Nasdaq and Tokyo Financial Exchange followed in 2018 (Bloomberg, 2017b; Tokyo Financial Exchange, 2017).

Coinbase Index (CBI).[3]

By 2017, Bitcoin, the first decentralised cryptocurrency that was created in 2008 and documented in Nakamoto (2008), had grown to a maximum of approximately 2,700% price return and, in the same year, some cryptocurrencies had achieved far higher growth than Bitcoin. Some economists, famous investors, and finance professionals warned that the rapidly increasing prices of cryptocurrencies could cause the "bubble" to burst. In fact, in early 2018, a large sell-off of cryptocurrencies occurred. From January to February 2018, the price of Bitcoin fell by 65%, and by the end of the first quarter of 2018, the entire cryptocurrency market fell by 54%, with losses in the market topping USD 500 billion. The decline of the cryptocurrency market was larger than the bursting of the Dot-com bubble in 2002. In November 2018, the total market capitalisation for Bitcoin fell below USD 100 billion for the first time since October 2017, and the Bitcoin price fell below 5,000 USD. More recently, the Bitcoin price has partially recovered and, in summer 2019, it traded at levels higher than 10,000 USD. As we can observe from Figure 1, such dynamics have been shared by all types of cryptocurrencies.

Cryptocurrency is the private sector counterpart of government-issued currency (Nakamoto, 2008; Ethereum, 2014; Ripple, 2012) and is issued in divisible units that can be easily transferred in a transaction between two parties. Digital currencies are intrinsically useless electronic tokens that travel through a network of computers. Advances in computer science have allowed for the creation of a decentralised system for transferring these electronic tokens from one person or firm to another. The key innovation of the cryptocurrency system is the creation of a payment system across a network of computers that does not require a trusted third party to update

[3]The CBI tracks the combined financial performance of all of the digital assets listed for trading in the US region by Coinbase. The components of the CBI are weighted by market capitalization, defined as price multiplied by supply.

balances and keep track of the ownership of the virtual units. The technology behind the system is called Blockchain.[4]

The characteristics of cryptocurrency are as follows. The first characteristic relates to the fact that cryptocurrency is not based on a central authority that holds private information. On the contrary, it relies on public information, such as computation, from a large number of individual distributed computers and servers connected to each other via the network and not by a recognised authority. Secondly, the issue of new currency and operations are validated by the network via complex pre-defined mathematical operations, an algorithm defined as proof of work. This kind of network approves pre-defined, encrypted and immutable operations, so history cannot be changed and manipulated. The last characteristic refers to the ease of payment and management. Cryptocurrency is, by definition, computer-based and when linked to a portfolio the only requirement for transferring value or paying bills is an internet connection.

Most previous studies have analysed cryptocurrency empirically. For example, Hencic and Gourieroux (2014) applied a non-causal autoregressive model to detect the presence of bubbles in the Bitcoin/USD exchange rate. Sapuric and Kokkinaki (2014) measured the volatility of the Bitcoin exchange rate against six major currencies. More recently, Catania et al. (2018) analysed and predicted cryptocurrency volatility, whereas Catania et al. (2019) predicted the full distribution of cryptocurrency. Both Bianchi (2018) and Giudici and Pagnottoni (2019) have investigated the structural relationships between cryptocurrency and other macroeconomic and financial time-series.

However, there have only been a few theoretical studies that have modelled cryptocurrency. In this regard, Boehme et al. (2015) introduced the economics, technology and governance of Bitcoin, whereas Fernández-

[4]Cryptocurrency is just one of the many applications of Blockchain.

Villaverde and Sanches (2019) developed a model of competition among privately-issued fiduciary currencies. Garratt and Wallace (2018) and Schilling and Uhlig (2019) focused on the exchange rate of Bitcoin and its theoretical determinants. Brunnermeier and Niepelt (2019) derived a model of money and liquidity in order to identify the sources of seignorage rents and liquidity bubbles in the context of cryptocurrency. As we will explain in the next section, most of these studies have assumed partial equilibrium models and did not examine the economic repercussions from the introduction of cryptocurrency to the overall economy and its different sectors.

We try to fill this gap and we develop a Dynamic Stochastic General Equilibrium (DSGE) where cryptocurrency is considered as the alternative to government currency. In our model, we assume that the utility function is non-separable across consumption and real balances of government currency and cryptocurrency in household preferences. Moreover, we include two separate demand shocks to government currency and cryptocurrency, respectively.

We estimate our model with Bayesian techniques using US and cryptomarkets monthly data for the period 2013:M6-2019:M3. Specifically we construct two new series to proxy the quantity of cryptocurrency and technological development, respectively. To the best of our knowledge, our work is the first attempt to provide a general equilibrium model with cryptocurrency and to estimate its parameters with Bayesian techniques.

Our estimated results confirm the assumption of non-separability between consumption and real balances of government currency and cryptocurrency. Comparing this data-driven finding with the related literature that allows assets or wealth to enter household preferences in an additive separable way (see, among others, Iacoviello, 2015 and Michaillat and Saez, 2019), we contribute to the ongoing debate concerning the nature of cryptocurrency by suggesting that cryptocurrency should be treated similarly to government currency. In this regard, our empirical results are in line with Gans and

Halaburda (2019) that have defined cryptocurrency as a private digital currency.[5]

Our empirical analysis indicates that the reaction of the economy to shocks in preferences, technology and monetary policy are in line with the findings of previous literature (see, for example, Ireland, 2004 and Andrés et al., 2009). In addition, the reaction of real balances of cryptocurrency is countercyclical to output in response to these shocks. Moreover, in response to technology and monetary policy shocks, we find a strong substitution effect between the real balances of government currency and the real balances of cryptocurrency.

Our results show that the economy responds differently to shocks in the demand for government currency and cryptocurrency. In particular, government currency demand shocks have larger effects on output, inflation and the nominal interest rate. We also find that cryptocurrency productivity shocks imply a fall in the nominal exchange rate between government currency and cryptocurrency. The increase in the productivity of cryptocurrency leads to lower real balances of government currency due to the substitution effect. In turn, both output and inflation fall, whereas the inflation rate increases. However, the magnitude of these effects is much lower than in the case of preference, technology and monetary policy shocks.

We are able to quantify the contributions of each shock in our model through a variance decomposition analysis. Our findings indicate that technology, preferences and monetary policy have the highest contribution in terms of variations in the key endogenous variables of our model. We also find that specific productivity shocks play an important role in the variation of real balances of cryptocurrency and nominal exchange rate

[5] An alternative strand of literature defines cryptocurrency as a cryptoasset or utility tokens, see Boreiko et al. (2019) for a review of the various definitions, because cryptocurrency is no yet full "money-like" due to its current limitations and has more uses than a form of money payment, including investment purposes (see https://www.bankofengland.co.uk/knowledgebank/what-are-cryptocurrencies).

between government currency and cryptocurrency.

Our analysis allows us to test for the role of the non-separability assumption between consumption and real balances of cryptocurrency. Our findings show that the functional form of the utility function matters in terms of the responses of several macroeconomic aggregates to cryptocurrency productivity shocks. In particular, a gain in cryptocurrency productivity induces stronger decreases in output and inflation in the case of non-separable utility function between consumption and real balances of cryptocurrency.

Finally, we assess the role of monetary policy in the presence of shocks to cryptocurrency productivity. Our sensitivity analysis indicates that the larger the response of the monetary policy rule to a change in government currency growth is, the stronger the decline in output.

Our study also provides two policy recommendations. Firstly, we show that an increase in cryptocurrency productivity has a negative effect on output. Therefore, the monetary authority could adjust its policy rate in response to changes in the real balances of cryptocurrency and include a weight for cryptocurrency growth in its policy reaction function. Secondly, we provide evidence that the response of the nominal interest rate to changes in government currency growth needs to be gradual in order for the central bank to mitigate the fall in output.

The remainder of this paper is organised as follows. Section 2 reviews the relevant literature. Section 3 outlines the new DSGE model on which our study is based. In Section 4, we present the data used for the analysis and our Bayesian estimates. Section 5 presents the impulse response functions based on our estimated model. In Section 6, we focus on the variance decomposition analysis. In Section 7, we assess the choice of utility function by distinguishing the cases of non-separable and separable household preferences between consumption and real balances of cryptocurrency. Section 8 provides a sensitivity analysis on different assumptions concerning the monetary policy rule. The concluding remarks are found in Section 9.

2 Literature review

Our paper refers to two different streams of literature. On the one hand, we contribute to studies that have developed theoretical models to analyse and describe cryptocurrency dynamics. However, these studies have focused mainly on partial equilibrium models. In our work, we develop a general equilibrium framework introducing cryptocurrency as an alternative to government currency. On the other hand, our study also contributes to the DSGE literature that has analysed the role of government currency in the economy.

Regarding the first strand of literature and the theoretical models, Boehme et al. (2015) presented the design principles and properties of Bitcoin's platform for a non-technical audience. They reviewed past, present, and future uses of Bitcoin, identifying risks and regulatory issues as Bitcoin interacts with the conventional financial system and the real economy.

Furthermore, Fernández-Villaverde and Sanches (2019) built a model of competition among privately-issued fiduciary currencies.[6] They found that the lack of control over the total supply of money in circulation has critical implications for the stability of prices across the economy. In other words, the economy ends up in a state of hyperinflation. These authors also showed that in the short and medium terms, the value of digital currencies goes up and down unpredictably as a result of self-fulfilling prophecies.

Another theoretical model analysing the exchange rate between fiat currency and Bitcoin was developed by Athey et al. (2016). In particular, they argued that the Bitcoin exchange rate can be fully determined by two market fundamentals: the steady-state transaction volume of Bitcoin when

[6]More specifically, Fernández-Villaverde and Sanches (2019) extended the Lagos and Wright (2005) model by including entrepreneurs who can issue their own currencies to maximise profits following a predetermined algorithm (as in Bitcoin).

used for payments, and the evolution of beliefs about the likelihood that the technology will survive. Garratt and Wallace (2018) also studied the behaviour of the Bitcoin-to-Dollar exchange rate. They used the model introduced by Samuelson (1958) with identical two-period lived overlapping generations with one good per date. After exploring the problems of pinning down money prices in the one-money model, these authors expanded their analysis to include a competing outside fiat money (Bitcoin), and they also discussed other aspects of competing cryptocurrencies.

More recently, Sockin and Xiong (2018) developed a model in which the cryptocurrency has two main roles: (*i*) to facilitate transactions of certain goods among agents; (*ii*) as the fee to compensate coin miners for providing clearing services for the decentralised goods transactions on the platform. As a consequence of the first role of cryptocurrency, households face difficulties in making such transactions as a result of severe search frictions. In turn, such rigidity induced by the cryptocurrency price leads to either no or two equilibria.

Schilling and Uhlig (2019) used a model in the spirit of Samuelson (1958), assuming that there are two types of money: Bitcoins and fiat money such as dollars. Both monies can be used for transactions. These authors found a "fundamental condition", which is a version of the exchange-rate indeterminacy result in Kareken and Wallace (1981), showing that the Bitcoin price in dollar terms follows a martingale, adjusted for the pricing kernel. Schilling and Uhlig (2019) also found that there is a "speculative condition", in which the dollar price for the Bitcoin is expected to rise, and some agents start hoarding Bitcoin in anticipation of the price increase. Finally, Brunnermeier and Niepelt (2019) developed a dynamic and stochastic model with heterogeneous households, firms, and banks, as well as the government sector. They demonstrated that a swap from public money to private money does not imply a credit crunch nor undermine financial stability.

However, most of the aforementioned theoretical studies have utilised partial equilibrium models. In our work, we develop a general equilibrium set-up. Many DSGE models have analysed the role of government currency in the economy. For example, Nelson (2002) presented empirical evidence for the US and the UK that real money base growth matters for real economic activity. In particular, they have shown that the presence of the long-term nominal rate in the money demand function increases the effect of nominal money stock changes on real aggregate demand when prices are sticky.

In addition, Christiano et al. (2005) developed a model embodying nominal and real rigidities that accounts for the observed inertia in inflation and persistence in output. They included money among the variables of interest and found that the interest rate and the money growth rate move persistently in opposite directions after a monetary policy shock.

A small monetary business cycle model which contains three equations summarising the optimising behaviour of the households and firms that populate the economy was developed by Ireland (2004). This author found that, if changes in the real stock of money have a direct impact on the dynamics of output and inflation, then that impact must come simultaneously through both the IS and the Phillips curve. In the same spirit, Andrés et al. (2009) have analysed the role of money in a general equilibrium framework focusing on the US and the EU. Their findings uncovered the forward-looking character of money demand.

Therefore, our work can be seen as an extension of these studies that redefines the standard DSGE model with money by including a new sector of the economy related to cryptocurrency, thereby generating endogenous supply and demand in a general equilibrium framework. In the next section, we present our structural model of the monetary business cycle with cryptocurrency in detail.

3 Model

3.1 Households

The representative household of the economy maximises the following expected stream of utility:

$$\max_{\{C_t, H_t, B_t, M_t^g, M_t^c\}} E\sum_{t=0}^{\infty} \beta^t A_t \left[u\left(C_t, \frac{\frac{M_t^g}{P_t}}{E_t^g}, \frac{\chi_t \frac{M_t^c}{P_t}}{E_t^c} \right) - \eta H_t \right] \quad (1)$$

where $0 < \beta < 1$ and $\eta > 0$. The budget constraint each period is given by:

$$M_{t-1}^g + \chi_{t-1} M_{t-1}^c + T_t + B_{t-1} + W_t H_t + D_t = P_t C_t + \frac{B_t}{R_t} + M_t^g + \chi_t M_t^c \quad (2)$$

The variable $\frac{M_t^g}{P_t}$ represents the real balance for government currency, whereas $\frac{M_t^c}{P_t}$ denotes the real balance for the cryptocurrency. Moreover, χ_t indicates the nominal exchange rate between the government currency and the cryptocurrency.

Equation (1) shows that consumption and real balances of government currency and cryptocurrency are non-separable. This non-separability makes the marginal utility of consumption a function of the amount of real balances of government currency and cryptocurrency optimally demanded by the households. Our approach implies that cryptocurrency is a private digital currency that is the alternative with respect to government currency.[7] Moreover, since cryptocurrency can be seen as an alternative currency that does not pay any interest, in equation (2) we assume that the representative household purchases cryptocurrency at $t-1$ in terms of government currency, $M_{t-1}^c = \frac{M_{t-1}^g}{\chi_{t-1}}$, and holds cryptocurrency at time t as $M_t^c = \frac{M_t^g}{\chi_t}$.[8]

[7] In particular, the fact that cryptocurrency enters into the utility function of the representative household implies that the government accepts it as a form of payment. Therefore, cryptocurrency can be used for any economic transaction and can be expressed in terms of government currency through the nominal exchange rate.

[8] In this regard, our modelling differs from standard open economy DSGE models with multiple currencies (see, among others, Bodenstein et al., 2011). In these models the exchange rate is used to convert the interest rate received by the representative household

In equations (1) and (2), C_t and H_t denote household consumption and labour supply during the period t. The shocks A_t, E_t^g and E_t^c follow the autoregressive processes:

$$\ln(A_t) = \rho^a \ln(A_{t-1}) + \varepsilon_t^a \tag{3}$$

$$\ln(E_t^g) = \rho^{eg} \ln\left(E_{t-1}^g\right) + \varepsilon_t^{eg} \tag{4}$$

$$\ln(E_t^c) = \rho^{ec} \ln\left(E_{t-1}^c\right) + \varepsilon_t^{ec} \tag{5}$$

where $0 < \rho^a, \rho^{eg}, \rho^{ec} < 1$ and the zero-mean, serially uncorrelated innovations ε_t^a, ε_t^{eg} and ε_t^{ec} are normally distributed with standard deviations σ^a, σ^{eg} and σ^{ec}. As we are going to show below, in equilibrium, shock A_t translates into disturbances to the model's IS curve, whereas E_t^g and E_t^c indicate disturbances to government money and cryptocurrency demand curves.

In the budget constraint, household sources of funds include T_t, a lump-sum nominal transfer received from the monetary authority at the beginning of period t, and B_{t-1}, the value of nominal bonds maturing during period t. The household's sources of funds also include labour income, W_tH_t, where W_t denotes the nominal wage, and nominal dividend payments, D_t, received from the intermediate goods-producing firms. The household's uses of funds consist of consumption, C_t, of finished goods, purchased at the nominal price, P_t, newly-issued bonds of value $\frac{B_t}{R_t}$, where R_t denotes the gross nominal interest rate.

It is convenient going forward to denote household real balances of government currency and cryptocurrency by $m_t^g = \frac{M_t^g}{P_t}$ and $m_t^c = \frac{M_t^c}{P_t}$, respectively. Moreover, we denote the gross inflation during period t with $\pi_t = \frac{P_t}{P_{t-1}}$.

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into holding foreign bonds. On the contrary, in our model the exchange rate allows for two currencies (i.e. government currency and cryptocurrency) that are used in the same economy to be converted.

3.2 Entrepreneurs

We assume that there is a continuum of entrepreneurs indexed by n, where $n \in [0,1]$, producing cryptocurrency. Each representative entrepreneur operates under a perfect competition. Following Sockin and Xiong (2018), we introduce a cost of producing cryptocurrency given by: $\kappa^{-\phi_t} Q_t^c$, where Q_t^c is the amount of tokens that the entrepreneur is producing. In addition:

$$\phi_t = \xi_t + \nu_t \tag{6}$$

is the entrepreneur's productivity, which depends on the productivity of the other entrepreneurs via the common component, ξ_t, as well as on the specific programming skills of the entrepreneur, ν_t. We assume that ξ_t and ν_t represent the common and specific productivity shocks to producing costs following the autoregressive processes:

$$\ln(\xi_t) = \rho^{\xi} \ln(\xi_{t-1}) + \varepsilon_t^{\xi} \tag{7}$$

$$\ln(\nu_t) = \rho^{\nu} \ln(\nu_{t-1}) + \varepsilon_t^{\nu} \tag{8}$$

where $0 < \rho^{\xi}, \rho^{\nu} < 1$, and the zero-mean, serially uncorrelated innovations, ε_t^{ξ} and ε_t^{ν}, are normally distributed with standard deviations σ^{ξ} and σ^{υ}. Entrepreneurs also gain a fraction $(1-\rho) \in (0,1)$ from selling the cryptocurrency to households at price $\frac{P_t}{\chi_t}$. Thus, entrepreneurs maximise their profits with respect to Q_t^c:

$$\Pi_t = \max_{\{Q_t^c\}} \left((1-\rho) \frac{P_t}{\chi_t} - \kappa^{-\phi_t} \right) Q_t^c \tag{9}$$

3.3 Production Goods Firms

We assume a continuum of monopolistically competitive firms indexed by $i \in [0,1]$ producing differentiated varieties of intermediate production goods, and a single final production good firm combining the variety of intermediate production goods under perfect competition. During each period $t = 0, 1, 2, ...$, the representative final goods-producing firm uses $Y_t(i)$ units of each intermediate good purchased at the nominal price, $P_t(i)$, to manufacture $Y_t(i)$ units of the final goods according to the constant-returns to-scale technology described by:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{(\theta-1)}{\theta}} \, di \right]^{\frac{\theta}{(\theta-1)}} \tag{10}$$

where $\theta > 1$. The final goods-producing firm maximises its profits by choosing:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta} Y_t \tag{11}$$

which reveals that θ measures the constant price elasticity of demand for each intermediate good. Competition drives the final goods-producing firm's profits to zero in equilibrium, determining P_t as:

$$P_t = \left[\int_0^1 (P_t(i))^{1-\theta} \, di \right]^{\frac{1}{1-\theta}} \tag{12}$$

During each period $t = 0, 1, 2, ...$, the representative intermediate goods-producing firm hires $H_t(i)$ units of labour from the representative household to manufacture $Y_t(i)$ units of intermediate good i according to the linear technology:

$$Y_t(i) = Z_t H_t(i) \tag{13}$$

where the aggregate productivity shock, Z_t, follows the autoregressive process:

$$\ln(Z_t) = \rho^z \ln(Z_{t-1}) + \varepsilon_t^z \tag{14}$$

where $0 < \rho^z < 1$, and the zero-mean, serially uncorrelated innovation, ε_t^z, is normally distributed with standard deviation σ^z. In equilibrium, this supply-side disturbance acts as a shock to the Phillips curve. Since the intermediate goods substitute imperfectly for one another in producing the final goods, the representative intermediate goods-producing firm sells its output in a monopolistically competitive market: the firm acts as a price-setter, but must satisfy the representative final goods-producing firm's demand at its chosen price. Similar to Rotemberg (1982), the intermediate goods-producing firm faces a quadratic cost of adjusting its nominal price, measured in terms of the final goods and given by:

$$\frac{\phi}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)}-1\right]^2 Y_t \tag{15}$$

with $\phi > 0$ and π measuring the gross steady-state inflation rate. This cost of price adjustment makes the intermediate goods-producing firm's problem dynamic: it chooses $P_t(i)$ for all $t = 0, 1, 2, ...$ to maximise its total market value. At the end of each period, the firm distributes its profits in the form of a nominal dividend payment, $D_t(i)$, to the representative household.

3.4 Monetary Policy

We assume that the central bank sets the nominal interest rate following a modified version of the Taylor (1993) rule given by:

$$\ln\left(\frac{R_t}{R}\right) = \rho^r \ln\left(\frac{R_{t-1}}{R}\right) + (1-\rho^r)\,\rho^y \ln\left(\frac{Y_t}{Y}\right) + (1-\rho^r)\,\rho^\pi \ln\left(\frac{\pi_t}{\pi}\right) + (1-\rho^r)\,\rho^{\mu^g} \ln\left(\frac{\mu_t^g}{\mu^g}\right) + \varepsilon_t^r \tag{16}$$

where:

$$\mu_t^g = \frac{\frac{M_t^g}{P_t}}{\frac{M_{t-1}^g}{P_{t-1}}} \tag{17}$$

In equation (16), ρ^r, ρ^y, ρ^π and ρ^{μ^g} are non-negative parameters, and the zero-mean, serially uncorrelated policy shock, ε_t^r, is normally distributed with the standard deviation σ^r. The monetary authority adjusts the short-term nominal interest rate in response to deviations of output and inflation from their steady-state levels as well as government currency growth as shown in equation (17). Andrés et al. (2009) have argued that an interest-rate rule that depends on the change in real balances of government currency may be motivated as part of an optimal reaction function when money growth variability appears in the central bank's loss function. As an alternative explanation, the response to money growth can be justified by money's usefulness in forecasting inflation.

3.5 Equilibrium

The symmetric equilibrium of the model can be log-linearised to obtain the following set of equations[9]:

$$\hat{y}_t = \hat{y}_{t+1} - \omega_1 \left(\hat{r}_t - \hat{\pi}_{t+1}\right) + \omega_2 \left[\left(\hat{m}_t^g - \hat{e}_t^g\right) - \left(\hat{m}_{t+1}^g - \hat{e}_{t+1}^g\right)\right] + \tag{18}$$

$$\omega_3 \left[\left(\hat{\chi}_t + \hat{m}_t^c - \hat{e}_t^c\right) - \left(\hat{\chi}_{t+1} + \hat{m}_{t+1}^c - \hat{e}_{t+1}^c\right)\right] + \omega_1 \left(\hat{a}_t - \hat{a}_{t+1}\right)$$

$$\hat{m}_t^g = \gamma_1 \hat{y}_t - \gamma_2 \hat{r}_t + \gamma_3 \hat{e}_t^g - \gamma_4 \hat{\chi}_t - \gamma_4 \hat{m}_t^c + \gamma_4 \hat{e}_t^c \tag{19}$$

$$\hat{m}_t^c = \gamma_5 \hat{y}_t - \gamma_6 \hat{r}_t + \gamma_7 \hat{e}_t^c - \gamma_8 \hat{\chi}_t - \gamma_8 \hat{m}_t^g + \gamma_8 \hat{e}_t^g \tag{20}$$

$$\hat{\pi}_t = \left(\frac{\pi}{R}\right) \hat{\pi}_{t+1} + \psi \begin{bmatrix} \left(\frac{1}{\omega_1}\right) \hat{y}_t - \left(\frac{\omega_2}{\omega_1}\right) \left(\hat{m}_t^g - \hat{e}_t^g\right) \\ - \left(\frac{\omega_3}{\omega_1}\right) \left(\hat{\chi}_t + \hat{m}_t^c - \hat{e}_t^c\right) - \hat{z}_t \end{bmatrix} \tag{21}$$

$$\hat{\chi}_t = -\varrho \hat{\phi}_t \tag{22}$$

$$\hat{\phi}_t = \left(\frac{\xi}{\phi}\right) \hat{\xi}_t + \left(1 - \frac{\xi}{\phi}\right) \hat{\nu}_t \tag{23}$$

$$\hat{r}_t = \rho^r \hat{r}_{t-1} + \left(1 - \rho^r\right) \rho^y \hat{y}_t + \left(1 - \rho^r\right) \rho^\pi \hat{\pi}_t + \left(1 - \rho^r\right) \rho^{\mu^g} \hat{\mu}_t^g + \varepsilon_t^r \tag{24}$$

Equation (18) represents a log-linearised version of the Euler equation that links the household's marginal rate of intertemporal substitution to the real interest rate. When ω_2 and ω_3 are different from zero, the household's utility function is non-separable across consumption and real balances of government currency and cryptocurrency. Since utility is non-separable, real balances of government currency and cryptocurrency affect the marginal rate

[9]The small letters with a hat, $\hat{x}_t$, denote the deviation of a given variable, X_t, from its steady-state value. The full derivation of the model together with the steady state solutions are shown in Appendix A.

of intertemporal substitution. Hence, additional terms involving $\hat{m}_t^g$ and $\hat{m}_t^c$ also appear in the IS curve.

Equation (19) takes the form of a money demand relationship for government currency, with income elasticity (γ_1), interest semi-elasticity (γ_2), elasticity of $\hat{m}_t^g$ with respect to government currency demand shocks (γ_3), and cross-elasticity with cryptocurrency (γ_4).[10] Moreover, equation (20) reveals the form of a money demand relationship for cryptocurrency, with income elasticity (γ_5), interest semi-elasticity (γ_6), elasticity of $\hat{m}_t^c$ with respect to cryptocurrency demand shocks (γ_7), and cross-elasticity with government currency (γ_8).[11]

Equation (21) is a forward-looking Phillips curve that also allows real balances of government currency ($\hat{m}_t^g$) and cryptocurrency ($\hat{m}_t^c$) to enter into the specification when ω_2 and ω_3 are non-zero. The non-separability in preferences across consumption and real balances of government currency and cryptocurrency implies a direct influence of the former variable on marginal cost and inflation; hence, real balances of government currency and cryptocurrency also appear in the Phillips curve.

Equations (18) and (21) also reveal that, wherever the real balances of government currency ($\hat{m}_t^g$) and cryptocurrency ($\hat{m}_t^c$) appear in the IS and Phillips curve relationships, they are followed immediately by the money demand disturbances, $\hat{e}_t^g$ and $\hat{e}_t^c$.

Equation (22) is the log-linearised first order condition derived from the profit maximisation problem of entrepreneurs that shows a negative relationship between the entrepreneurs' productivity and the exchange

[10]In equation (19), we note that an increase in the demand for cryptocurrency decreases the real balances of government currency. In Section 4.4, we will show that the estimated value of the cross elasticity of government currency demand and cryptocurrency demand is high.

[11]Equation (20) indicates that the real balances of cryptocurrency decrease when the demand of government currency rises. In Section 4.4, our estimated results will show that the value of γ_8 is above unity, indicating a strong substitution effect between cryptocurrency and government currency.

rate between government currency and cryptocurrency. This expression reflects the well-established feature of cryptocurrency that is based on a cryptographic proof-of-work system. Such system relies on solving complex mathematical operations and generating new coins with this validation. The process is known as mining. Nakamoto (2008) proposed a largest pool of central processing unit (CPU). However, since 2011, a graphical processing unit (GPU) has revolutionised the computational power and its introduction has stimulated the diffusion of Bitcoin. In our model, this mechanism can be interpreted as a positive productivity shock for the mining of cryptocurrency, reducing the exchange rate value between government currency and cryptocurrency.

Equation (23) is the log-linearised expression for the entrepreneurs' productivity that depends on the common productivity in the cryptocurrency sector as well as on the specific productivity of the entrepreneur. Equation (24) shows the log-linearised relation for the monetary policy rule, indicating that the interest rate adjusts to output, inflation and government currency growth.

The cryptocurrency market is in equilibrium if the quantity of cryptocurrency supplied by entrepreneurs is equal to the demand of cryptocurrency by households. The goods market clearing condition implies that the output produced by production goods firms is equal to households' consumption. The model is closed by adding the log-linearised versions of the AR(1) processes for the preferences shock to consumption, the demand shocks for government currency and cryptocurrency, the common and specific productivity shocks of cryptocurrency as well as the aggregate technology shock.

4 Estimating the Model

In this section, we estimate the model described in Section 3 using Bayesian techniques. In what follows, we initially describe the data used in order to estimate the model (Section 4.1). Successively, we present the parameters of the model (Section 4.2) and their identification (Section 4.3). Finally, we describe the estimation results (Section 4.4).

4.1 Data

The main challenge in estimating our model is the relatively short sample for the macroeconomic series related to the market of cryptocurrency due to its recent development. Accordingly, in order to have a sufficient number of observations for our estimated model, we decided to use US data at monthly frequency. Foroni and Marcellino (2014) have dealt with DSGE models estimated with mixed frequency data, including monthly data. Our sample period corresponds to 2013:M6-2019:M3. We use seven data series in the estimation because there are seven shocks in the theoretical model (see Table 1).[12]

The seven data series include the industrial production index, the natural log of real private consumption, the natural log of real money stock, the real Bitcoin price, the real cumulative initial coin offering (ICO), the real Nvidia volume weighted average price and the effective federal funds rate. All the real variables are deflated by the consumer price index (CPI). Real private consumption and real M2 money stock are expressed in per capita terms, divided by working-age population.

Focusing on monetary variables, we follow Ireland (2004) by considering money stock M2 as an indicator that includes a broader set of financial assets held principally by households. The real Bitcoin price is obtained

[12]The data sources and the construction of all observed variables are reported in Appendix B.

from the monthly average of daily data, assuming that the daily price is the average between opening and closing prices. We consider the Bitcoin price to be representative of the cryptocurrency price. Our choice is related to the longer sample period that is available for the Bitcoin price compared to the CBI. Our assumption is plausible since, for the same sample period, the correlation between the CBI and the Bitcoin price corresponds to 99%.

The ICO or initial currency offering is a type of funding which uses cryptocurrency. In an ICO, a quantity of cryptocurrency is sold in the form of tokens to speculator investors, in exchange for legal tender or another cryptocurrency. The tokens sold are promoted as future functional units of currency if the ICO's funding goal is met and the project is launched. The Nvidia volume weighted average price is obtained as a monthly average from daily data. The Nvidia Corporation is the most important American technology company that designs graphics processing units (GPUs) for the gaming and professional markets, as well as systems on chip units (SoCs) for the mobile computing and automotive market.

4.2 Model Parameters

We decided to split the parameters of the model into two groups. The first group of parameters is fixed and consistent with data at a monthly frequency. In line with Ireland (2004), we assume ω_1 equal to one, implying the same level of risk aversion as a utility function that is logarithmic in consumption. The parameter ψ is fixed equal to 0.1 following King and Watson (1996), Ireland (2000) and Ireland (2004). Such a value implies that the fraction of the discounted present value and future discrepancies between the target price and the actual price of production goods is equal to 10%. The steady-state values for the nominal interest rate and inflation are computed from the monthly data of the effective federal funds rate and natural log changes in CPI. For our sample period they are equal to 0.70% and 0.13%, respectively.

The second group of parameters is estimated with the Bayesian technique (Tables 2 and 3). To the best of our knowledge, our study is the first attempt to estimate a DSGE model that includes cryptocurrency. Hence, this is one of our main contributions and we rely on our judgement and the findings of previous DSGE models that consider government currency (e.g., Ireland, 2000, Ireland, 2004 and Andrés et al., 2009).

Table 2 shows the prior distributions for the endogenous parameters of our model. For the parameter indicating the output elasticity with respect to real balances of government currency (ω_2), we assume that its prior mean is in line with the range of estimates by Ireland (2004). On the other hand, we assume that the prior mean of the elasticity of output with respect to real balances of cryptocurrency (ω_3) is one fourth lower than that of government currency.

In order to set up the priors for the income elasticity of government currency demand (γ_1), the interest semi-elasticity of government currency demand (γ_2) and the elasticity of real balances of government currency with respect to government currency demand shocks (γ_3), we follow the estimated results of Ireland (2004) for the US economy. Moreover, we assume a prior mean value for γ_4 such that changes in the demand for cryptocurrency can affect the real balances of government currency.

Focusing on the parameters that characterise the demand relationship for cryptocurrency, we assume that γ_6 has a higher prior mean value than γ_5. Moreover, we assume that the real balances of cryptocurrency are strongly affected by exogenous changes in cryptocurrency demand, which corresponds to a large prior mean for γ_7. Moreover, following Gans and Halaburda (2019), we believe that cryptocurrency is a valuable alternative to government currency and assume a high prior mean value for γ_8.

In line with Athey et al. (2016) and Garratt and Wallace (2018), we acknowledge that the exchange rate between government currency and cryptocurrency is an important determinant of the cryptocurrency productivity and, in turn, we assume a high prior mean value for ϱ.

Turning to the parameter measuring the relative importance of common productivity with respect to specific productivity in the production of cryptocurrency ($\frac{\xi}{\phi}$), we are agnostic about its prior and, in turn, we assume that it covers a reasonable range of values.

Regarding the parameters of the monetary policy rule, the prior for the degree of interest rate smoothing (ρ^r), the reaction coefficient of output (ρ^y), the interest-rate response to inflation (ρ^π) and government currency growth (ρ^{μ^g}) are all in line with the estimates by Andrés et al. (2009).

Table 3 reports the priors of the parameters related to the exogenous processes driving the economy. We set the persistence parameters of all autoregressive exogenous processes to be Beta distributed. We assume that the technology shock is more persistent than consumption preference and government currency demand shocks. We also assume that the prior for the persistence of the cryptocurrency demand shock has a relatively low value. For both productivity shocks to cryptocurrency, we assume that their prior means and standard deviations correspond to 0.60 and 0.05, respectively. Finally, we use Inverse Gamma distributions for standard errors of all exogenous shocks with means equal to 0.01 and infinite degrees of freedom which correspond to rather loose priors.

4.3 Parameter Identification

We estimated our model using a sample of 5,000,000 draws and we dropped the first 1,250,000.[13] Our acceptation rate corresponds to 37%. In order to test the stability of the sample, we used the Brooks and Gelman (1998) diagnostics test, which compares within and between moments of multiple chains. Moreover, we performed other diagnostic tests for our estimates, such as the Monte Carlo Markov Chain (MCMC) univariate diagnostics and the

[13] In order to perform our estimation analysis, we used Dynare (http://www.dynare.org/).

multivariate convergence diagnostics.[14]

As it is well known, the lack of identification in the parameter values is a potentially serious problem for the quantitative implications of DSGE models (see, for example Canova and Sala, 2009). Accordingly, we compared the prior and posterior distributions of the model parameters. For most of the parameters we found that prior probability density functions are wide, and posterior distributions are different from the priors.[15] Moreover, we performed the test proposed by Iskrev (2010).[16] This test checks the identification strength and sensitivity component of the parameters using a rank condition based on the Fischer information matrix and the moment information matrix normalised by either the parameter at the prior mean or by the standard deviation at the prior mean. Our results show that the derivative of the vector of predicted autocovariogram of observables with respect to the vector of estimated parameters has full rank when we evaluate it at the posterior mean estimate. This implies that all the parameters are identifiable in the neighbourhood of our estimates.

4.4 Estimated Results

Tables 2 and 3 show the posterior means for the endogenous and exogenous parameters with their 90% confidence intervals.

We start by focusing on the estimated parameters of the IS curve. From Table 2, we note that the estimated posteriors of ω_2 and ω_3 are different from zero. This result indicates that the utility function is non-separable between consumption and real balances of government currency and cryptocurrency. Interestingly, the estimated values ω_2 and ω_3 imply that the output response to changes in real balances of government currency is more than six times

[14]The plots for MCMC univariate and multivariate convergence diagnostics are shown in Appendix C.

[15]We report the plots for prior and posterior density functions of all parameters in Appendix C.

[16]The plots showing the results for this test are reported in Appendix D.

higher compared to variations in real balances of cryptocurrency. As we will see in the next section, this result has important consequences for the effects of cryptocurrency productivity shocks on the economy.

Turning to the parameters of the money demand equation for government currency, our estimated values of γ_1, γ_2 and γ_3 are in line with the ranges of estimates provided by Ireland (2004), implying that the demand shock ($\hat{e}_t^g$) has the highest influence on the movements in the real balances of government currency. Moreover, the estimated posterior of γ_4 is well identified and indicates an important degree of substitution between the demand of government currency and cryptocurrency.

Now we focus on the estimated parameters included in the money demand equation for cryptocurrency. From Table 2, it is possible to note that the posterior mean of γ_6 is much higher than γ_5, implying that real balances of cryptocurrency respond more to changes in nominal interest rate than to variations in output. As will be shown in Section (5), this result has an important effect in terms of the response of cryptocurrency demand to the preference shock. Moreover, we find that the posterior means of γ_7 and γ_8 are above unity. These results have two main implications. Firstly, they suggest that the demand shock ($\hat{e}_t^c$) plays a substantial role in terms of variation in the real balances of cryptocurrency. Secondly, our estimates indicate a strong elasticity of substitution between cryptocurrency and government currency. This result will be discussed further in the next section. In particular, we are going to show that the change in government currency demand greatly affects the demand for cryptocurrency.

Focusing on the parameters related to the production of cryptocurrency, the estimated posterior of ϱ is well identified and has a value slightly below unity. Our result confirms the studies by Garratt and Wallace (2018) and Athey et al. (2016) who found that the exchange rate between government currency and cryptocurrency is an important determinant of cryptocurrency production. Moreover, the estimated value of $\frac{\xi}{\phi}$ suggests that common productivity has a stronger impact than the specific productivity in terms of cryptocurrency production. This implies that common productivity shocks have larger effects on the economy than specific productivity shocks.

Turning to the estimates of the monetary policy reaction function, we observe that in our sample period there is significant interest-rate smoothing. In addition, the nominal interest rate appears to react more strongly to variations in the inflation rate than to output changes. Interestingly, our estimated parameter for the interest-response to government currency growth (ρ^{μ^g}) has a higher value than in Andrés et al. (2009). This result suggests that the central bank relies on the government currency growth to set up its policy rate.

Table 3 shows the posterior estimates for the exogenous processes. In general, the posteriors of these parameters are well identified. We note that technology and preference shocks are more persistent than government currency and cryptocurrency demand shocks. Moreover, we find that the specific productivity shock to cryptocurrency production is slightly more persistent than the common productivity shock to cryptocurrency production. Finally, our posterior estimates show that shocks to specific cryptocurrency productivity, cryptocurrency and government currency demand are much more volatile than the remaining shocks.

5 Impulse Response Functions

In this section, we show the results of impulse response functions (IRFs) for the estimated model and consider some of the exogenous shocks driving the economy. Firstly, we focus on the "traditional" shocks to preferences, technology and monetary policy. Secondly, we analyse the shocks to the demand of households for real balances of government currency and cryptocurrency. Finally, we consider the "new" shocks to cryptocurrency common and specific productivity. We consider a positive 1% shock for each of these exogenous processes and we set the values of the estimated parameters of the model equal to their mean estimates of the posterior distribution.[17]

5.1 "Traditional" Shocks

Figures 2–4 present the responses of output, real balances of government currency and cryptocurrency, nominal exchange rate between government currency and cryptocurrency, inflation rate, and nominal interest rate.

From Figure 2, we note that, on impact, the preferences shock increases output and inflation by about 0.6% and 0.1%, respectively. The monetary authority responds by increasing the nominal interest rate that achieves its peak after two months. On impact, the real balances of government currency increase but, after only two months, they fall, exhibiting a strong inverse relationship with the nominal interest rate. These results are in line with the findings by Ireland (2004) and Andrés et al. (2009). Focusing on the real balances of cryptocurrency, we observe that they decrease in response to this shock.[18] This result is a consequence of the larger estimated value for the interest semi-elasticity (γ_6) than the income elasticity of cryptocurrency demand (γ_5). Finally, we observe that the response of the nominal exchange rate between government currency and cryptocurrency remains almost unchanged in response to the preferences shock.

Figure 3 shows the IRFs for the technology shock. We find that a 1% positive shock to technology increases output and the peak is achieved after seven months and corresponds to about 0.97%. Inflation decreases on impact by about 0.16% and it remains negative for all the periods considered in the graph. Accordingly, the monetary authority decreases its policy rate. Real balances of government currency exhibit an inverse relationship with

[17]Accordingly, our strategy allows us to compare the impulse responses among the different shocks. In Appendix E, we present the estimated impulse responses together with their confidence intervals.

[18]On impact, the demand of cryptocurrency drops by 0.01%.

the nominal interest rate and have their peak response seven months after the occurrence of the shock. These findings are in line with the results reported by Ireland (2004) and Andrés et al. (2009). Furthermore, we observe a strong substitution effect between the real balances of cryptocurrency and government currency. This result is a consequence of the large estimated value for cross elasticity of cryptocurrency demand and government currency demand (γ_8). Finally, our results indicate that the nominal exchange rate between government currency and cryptocurrency is not affected by the technological shock.

Figure 4 shows that a positive shock of 1% to monetary policy induces an increase in the nominal interest rate by 0.7%. In response to the shock, both output and inflation fall.[19] The negative response of output and the positive response of nominal interest rate induce the fall in the demand for government currency. These results confirm the findings of Ireland (2004) and Andrés et al. (2009). Moreover, our results suggest a strong substitution effect between real balances of cryptocurrency and government currency, with the former increasing by 0.02% on impact. However, the impulse response of the nominal exchange rate between government currency and cryptocurrency shows a negligible change.

Our interesting and novel results indicate that when cryptocurrency is considered in the economy as an alternative currency option, we observe a strong substitution effect between real balances of cryptocurrency and government currency. In particular, our estimated model suggests that real balances of cryptocurrency are countercyclical to output, whereas government currency is procyclical in response to preferences, technology and monetary shocks.

[19] On impact, output decreases by 2.3% and inflation by 0.5%.

5.2 Government Currency Demand Shocks vs. Cryptocurrency Demand Shocks

Figure 5 presents the impulse responses to real balances of government currency (blue lines) and cryptocurrency demand shocks (red lines).

The positive shock on government currency demand induces both real balances of government currency and cryptocurrency to rise. This result is a consequence of the large estimated values of γ_3 and γ_8. As described above, in the IS and Phillips curves the real balances of government currency are immediately followed by the government currency demand shock.[20] Since the response of the shock to government currency demand is systematically higher than that of the real balances of government currency, output decreases and inflation rate increases.

Furthermore, we find that the nominal interest rate drops in response to this shock. This may be explained by the fall in the government currency growth that induces the central bank to decrease its policy rate. Finally, we observe that the nominal exchange rate between government currency and cryptocurrency does not move in response to the shock.

Now we focus on the effects of a positive shock to cryptocurrency demand. We begin by noticing that the real balances of cryptocurrency increase in response to this shock. Moreover, because of the large estimated value of γ_7, the positive response of real balances of cryptocurrency is systematically higher than the shock to cryptocurrency demand. This implies that the real balances of government currency fall.

From Figure 5, we also observe that the effects of this shock on output, inflation and nominal interest rate are weak.[21] This finding can be explained by the low estimated value of ω_3. In particular, on the impact of the shock, output increases, whereas inflation rate falls from the second month onwards.

[20]In particular, equations (18) and (21) show a difference between the real balances of government currency and the government currency demand disturbance.

[21]On impact, output increases by only 0.01%.

Moreover, the increase in government currency growth leads the central bank to raise its policy rate. Also in this case, the response of the exchange rate between government currency and cryptocurrency is almost unchanged.

Overall, the above results indicate that shocks to government currency demand have larger spillover effects on the economy than shocks to cryptocurrency demand. To the best of our knowledge, this is the first time that such a result has been documented in a general equilibrium framework and it is mainly driven by our key estimated parameters.

5.3 Shocks to Cryptocurrency Productivity

The shocks to common and specific productivity of cryptocurrency are presented in Figure 6. The impulse responses to the former shock are shown in blue lines, whereas the impulse responses to the latter shock are in red lines.[22]

In general, a positive shock to the productivity of entrepreneurs producing cryptocurrency implies a fall in the nominal exchange rate between government currency and cryptocurrency.[23] The decrease in the exchange rate induces an increase in the real balances of cryptocurrency.[24] The demand of government currency drops as a consequence of the substitution effect with cryptocurrency demand. However, we note that, in terms of magnitude, the fall in the real balances of government currency is much lower than the increase in the real balances of cryptocurrency.[25]

From Figure 6, we note that output falls in response to cryptocurrency productivity shocks. This result is a consequence of the larger estimated value

[22] Although the magnitude of effects of common and specific productivity shocks differ, the responses of the several macroeconomic variables to these shocks are qualitatively the same.

[23] On impact, the nominal exchange rate falls by 0.45% and 0.33% in response to the common and the specific shocks, respectively.

[24] In the case of a common shock, the increase corresponds to 0.46%, whereas it is equal to 0.34% for the specific shock.

[25] The government currency demand decreases only by 0.006% and 0.005% in response to common and specific shocks, respectively.

of the output elasticity to the real balances of government currency (ω_2) than the output elasticity with respect to cryptocurrency (ω_3). Our findings also indicate that the change in output in response to these shocks is much less pronounced than in the case of the "traditional" shocks.[26] The reduction in aggregate output induces a decrease in the inflation rate. Moreover, the increase in government currency growth leads the central bank to raise the nominal interest rate. We note that, in terms of magnitude, the changes in both inflation and nominal interest rate are negligible compared to their responses in the case of "traditional" shocks.[27]

To summarise, the common and specific productivity shocks generate qualitatively similar reactions to the economy. In particular, the nominal exchange rate decreases due to the higher cryptocurrency productivity. This leads to lower real balances of government currency, due to the substitution effect, which in turn reduces the inflation rate. However, the impact to the economy from these shocks is not as strong in comparison to the "traditional" shocks presented earlier.

6 Variance Decomposition Analysis

Table 4 shows the importance of each shock in terms of fluctuations in the key endogenous variables of the model. In particular, the variance decomposition analysis is based on the simulation of the estimated model (10,000 iterations).[28] More specifically, our strategy consists of two steps. As a first step, we run the model estimation and we obtain that the parameters and the variance matrix of the shocks are set to the mode for the maximum

[26] Common and specific productivity shocks induce a fall in output of only 0.002% and 0.001%, respectively.

[27] On impact, both common and specific productivity shocks induce a fall in the inflation rate of only 0.0002%. The peak responses of nominal interest rate to common and specific productivity shocks are 0.00002% and 0.00001%, respectively.

[28] Our simulation results are detrended using the HP filter with a smoothing parameter equal to 1,600.

likelihood estimation or posterior mode computation. As a second step, we simulate the model so that our simulation of the estimated model is based on the posterior modes of the model.[29]

In Table 4, we observe that "traditional" shocks explain most of the variations in output and inflation. In particular, the contributions of technology shocks on both output and inflation changes are almost 90%. The "traditional" shocks also have an important influence on the nominal interest rate. More specifically, 89% of the variation in the nominal interest rate is explained by a combination of technology and monetary policy shocks. The remaining 11% is explained by preferences, government currency and cryptocurrency demand shocks.

As expected, our results also show that government currency and cryptocurrency demand shocks contribute to most of the variations in the real balances of government currency and cryptocurrency (98% and 84%, respectively). Moreover, we find that the shock to cryptocurrency specific productivity accounts for 13% in terms of variation in the real balances of cryptocurrency. Interestingly, the variation in the nominal exchange rate between government currency and cryptocurrency is almost entirely explained by shocks to cryptocurrency specific productivity.

These results are confirmed by the forecast error variance decomposition, which we show for 1, 5, 12 and 30 periods ahead (Table 5). The "traditional" shocks (technology, preferences and monetary policy) have the highest contribution in terms of variations in the key endogenous variables of our model. We also find that specific productivity shocks play an important role in the variation of cryptocurrency demand and exchange rate between government currency and cryptocurrency.

[29]In general, it is preferable to follow this approach because the exact distributions of the posteriors are not known. Consequently, in the presence of irregular posteriors the mode is preferred to the mean as a measure of the central tendency of the distribution.

7 How Can Cryptocurrency Be Introduced into the Utility Function?

In Section 4, our estimated results indicated that ω_3 is different from zero. This result confirms the assumption that the utility function is non-separable between consumption and the real balances of cryptocurrency. In turn, this implies that the marginal utility of consumption is a function of the amount of these real balances optimally demanded by households. Therefore, a change in the real balances of cryptocurrency has a direct positive impact on household consumption. As explained in Section 3, the non-separability assumption introduces terms involving the real balances of cryptocurrency into the IS and the Phillips curves.[30] Hence, in equilibrium, output and inflation depend on the current and expected real balances of cryptocurrency after accounting for cryptocurrency demand shocks.

In this section, we are going to provide a counterfactual analysis about this non-separability assumption. Our experiment relates to the ongoing debate concerning the nature of cryptocurrency. As we explained above, several studies (see, for example, Gans and Halaburda, 2019 and Boreiko et al., 2019) view cryptocurrency as a private digital currency that is the alternative to government currency. In our model, this can be obtained by assuming that real balances of cryptocurrency are non-separable from real balances of government currency and consumption in the utility function, as our estimated model indicates.

On the other hand, Rohr and Wright (2019) have argued that cryptocurrency can be interpreted as an asset, also defined as a digital token. Recent literature (see, for example, Iacoviello, 2015 and Michaillat and Saez, 2019) has assumed that assets or wealth enter households' preferences in an additive separable way. In order to incorporate these arguments into our

[30] In equations (18) and (21), these additional terms are the shift-adjusted real balances of cryptocurrency, i.e., $\hat{\chi}_t + \hat{m}_t^c - \hat{e}_t^c$, respectively.

model, we test the case in which the utility function is separable between the real balances of cryptocurrency and consumption. In this case, the marginal utility of consumption is independent of the real balances of cryptocurrency. As a result, we expect that cryptocurrency affects household consumption only indirectly via the wealth effect from holding this asset.

Practically, the effect of the real balances of cryptocurrency on aggregate demand vanishes when the parameter ω_3 is equal to zero, i.e., as long as the cross derivative between consumption and real balances of cryptocurrency is zero in the utility function. Therefore, we re-estimate our model assuming that ω_3 is equal to zero. We further extend this analysis by considering the case of separability between consumption and the real balances of government currency and cryptocurrency. Thus, we also re-estimate our model, assuming that both ω_2 and ω_3 are equal to zero. This experiment provides a more direct comparison of our approach with the introduction of cryptocurrency and the work of Ireland (2004) and Andrés et al. (2006) that incorporate a separable utility function between private consumption and real balances of government currency.

Table 6 reports the comparison between the estimated results of the benchmark model (with non-separable utility function), counterfactual model A (with separable utility function between consumption and the real balances of government currency and cryptocurrency, i.e., $\omega_2 = 0$ and $\omega_3 = 0$) and counterfactual model B (with separable utility function between consumption and the real balances of cryptocurrency, i.e., $\omega_3 = 0$) for the endogenous parameters, showing their posterior mean estimates and 90% confidence intervals.[31]

Our results indicate that, in the counterfactual model B, the estimated value of ω_2 substantially increases with respect to the benchmark model (from 0.20 to 0.24). This finding implies that changes in the real balances

[31] In Appendix F, we report the estimated values of the exogenous parameters together with their 90% confidence intervals.

of government currency have a stronger impact on output and inflation. In line with Ireland (2004) and Andrés et al. (2009), we find that the estimated value for γ_2 changes between the benchmark model and counterfactual model A. We extend these studies by showing that changes in the functional form of the utility function has also important effects on the estimated value for γ_3, γ_4, γ_7, ϱ and $\frac{\xi}{\phi}$. This means that our estimation procedure compensates for the elimination of cryptocurrency and/or government currency from the IS and the Phillips curves with different values of these parameters in order to regulate the movement in the shift-adjusted real balances of government currency and cryptocurrency.[32]

In order to better understand the main transmission mechanisms at work, we compare the IRFs of the benchmark model with those of the counterfactual models. Figure 7 shows the responses of the several macroeconomic aggregates to cryptocurrency common and specific productivity shocks.[33] The most important message of these figures is that the functional form of the utility function matters. Indeed, our results indicate that there is a much more pronounced decrease in output for the model with the non-separable utility function compared to the models with separable utility function.[34]

Interestingly, in counterfactual model B, inflation has a smaller decrease than in the benchmark model. This means that the introduction of the

[32]The shift-adjusted real balances of government currency and cryptocurrency are $\hat{m}_t^g - \hat{e}_t^g$ and $\hat{\chi}_t + \hat{m}_t^c - \hat{e}_t^c$, respectively.

[33]As above, we simulate cryptocurrency common and specific productivity gains of 1%. In Appendix F, we report the IRFs for the "traditional" shocks (i.e., preferences, technology and monetary policy shocks) as well as government currency and cryptocurrency demand shocks. For these shocks, our main findings are in line with the related literature that examines the impact of government currency in the utility function (see, for example, Ireland, 2004 and Andrés et al., 2006).

[34]As shown in equation (18), in counterfactual model A, real balances of government currency and cryptocurrency do not enter the IS curve. Differently, in counterfactual B, only real balances of cryptocurrency do not enter the IS curve. Therefore, the response of output is relatively more muted in counterfactual A compared to counterfactual B (and, in turn, compared to benchmark), even if inflation and nominal interest rate in counterfactual A do not deviate much from the benchmark.

real balances of cryptocurrency in the utility function in a non-separable way induces a stronger response of inflation to cryptocurrency productivity shocks. Accordingly, in the benchmark model there is a stronger reaction of the interest rate compared to counterfactual model B.

We also note that, in the counterfactual models, the response of the real balances of government currency is lower than in the benchmark model. This can be explained by the smaller estimated value of γ_4 in these models.[35] Finally, we note that the responses of the several macroeconomic aggregates to the two cryptocurrency productivity shocks are similar, but the effects of the cryptocurrency common productivity shock have a stronger intensity compared to the cryptocurrency specific productivity shock.

8 Different Assumptions about the Taylor Rule

In this section, we investigate the role of monetary policy in the presence of the shocks to cryptocurrency productivity. In particular, we provide a sensitivity analysis with three different scenarios of the Taylor rule (24). More specifically, the parameter measuring the response of the policy rate to government currency growth (ρ^{μ^g}) is assumed to be: equal to its estimated value (benchmark scenario), equal to zero (scenario 1),[36] and equal to the double of its estimated value in our model (scenario 2).[37]

Figures 8 and 9 show the responses of the key variables of our model in the cases of cryptocurrency common and specific productivity shocks, respectively.[38] The solid lines represent the impulse responses of the variables

[35]This implies a smaller increase in government currency growth in the counterfactual models.

[36]This assumption implies no weight of government currency growth in the Taylor rule.

[37]This assumption implies a higher weight of government currency growth in the Taylor rule.

[38]As above, we simulate a 1% increase in common and specific productivity of cryptocurrency.

in the benchmark scenario, whereas the dashed and dotted lines show the impulse responses for the same variables in scenarios 1 and 2, respectively.

In general, the increase in the entrepreneurs' productivity induces a drop in the nominal exchange rate between government currency and cryptocurrency. Accordingly, the real balances of cryptocurrency increase. We also observe a substitution effect between cryptocurrency demand and government currency demand with a reduction of the latter. As explained above, these effects induce output to fall. However, from Figures 8 and 9, we note that the magnitude of this decrease is different between the three scenarios. This result clearly depends on the response of the central bank to cryptocurrency (common and specific) productivity shocks. When the monetary authority does not consider government currency growth in the Taylor rule (scenario 1), the nominal interest rate falls. In turn, the decreases in output and inflation are less pronounced than in the benchmark case. On the contrary, when the weight of government currency growth in the Taylor rule is higher (scenario 2), the increase in the nominal interest rate is larger than in the benchmark case. In turn, this effect induces a larger decrease in output and inflation.

The different magnitude of the fall in output between the three alternative scenarios also has consequences on the inflation rate. As can be observed from Figures 8 and 9, in scenario 2, inflation falls more than in the benchmark case whereas, in scenario 1, it slightly increases.[39]

9 Conclusion

In this paper, we have developed and estimated a Dynamic Stochastic General Equilibrium (DSGE) model to evaluate the economic repercussions of cryptocurrency. Our model assumed that the representative household

[39]The small increase in inflation in scenario 1 is also due to the fall in the nominal interest rate.

maximises its utility, by also accounting for cryptocurrency holdings. Moreover, in our theoretical framework, we included entrepreneurs that determine the supply of cryptocurrency in the economy. We estimated our model using US monthly data and we compared our empirical findings with the "state-of-art" models without cryptocurrency.

We provided an impulse response analysis to show the effects of preferences, technology and monetary policy shocks on the real balances of government currency, as well as to the real balances of cryptocurrency. Moreover, we evaluated the responses of the main macroeconomic fundamentals to productivity shocks for the production of cryptocurrency.

We found a strong substitution effect between the real balances of government currency and the real balances of cryptocurrency in response to technology, preferences and monetary policy shocks. Moreover, government currency demand shocks had larger effects on the economy than shocks to cryptocurrency demand. We also found that cryptocurrency productivity shocks imply a fall in the nominal exchange rate. Output and inflation fall, whereas the nominal interest rate increases. However, the magnitude of the effects of these shocks was much lower than the "traditional" shocks.

Overall, our work provides novel insights and new evidence on the underlying mechanisms of cryptocurrency and the spillover effects it has on the economy. This can provide guidance to investors, policy makers, central bankers and researchers on how to act towards cryptocurrency and its ecosystem in the future. In particular, two policy recommendations emerge from our analysis. Firstly, we have shown that an increase in cryptocurrency productivity has a negative effect on output and inflation. Therefore, the monetary authority could decide to adjust its policy rate in response to changes in the real balances of cryptocurrency, including a weight for cryptocurrency growth, in its policy reaction function. Secondly, we provided evidence that the response of the nominal interest rate to changes in government currency growth needs to be gradual if the central bank wants to avoid a fall in output.

Our analysis opens up several extensions. For example, our estimated DSGE framework could be extended to a two-country exercise, extending studies on global cryptocurrency, such as Benigno et al. (2019), or even to a heterogeneous household setup.

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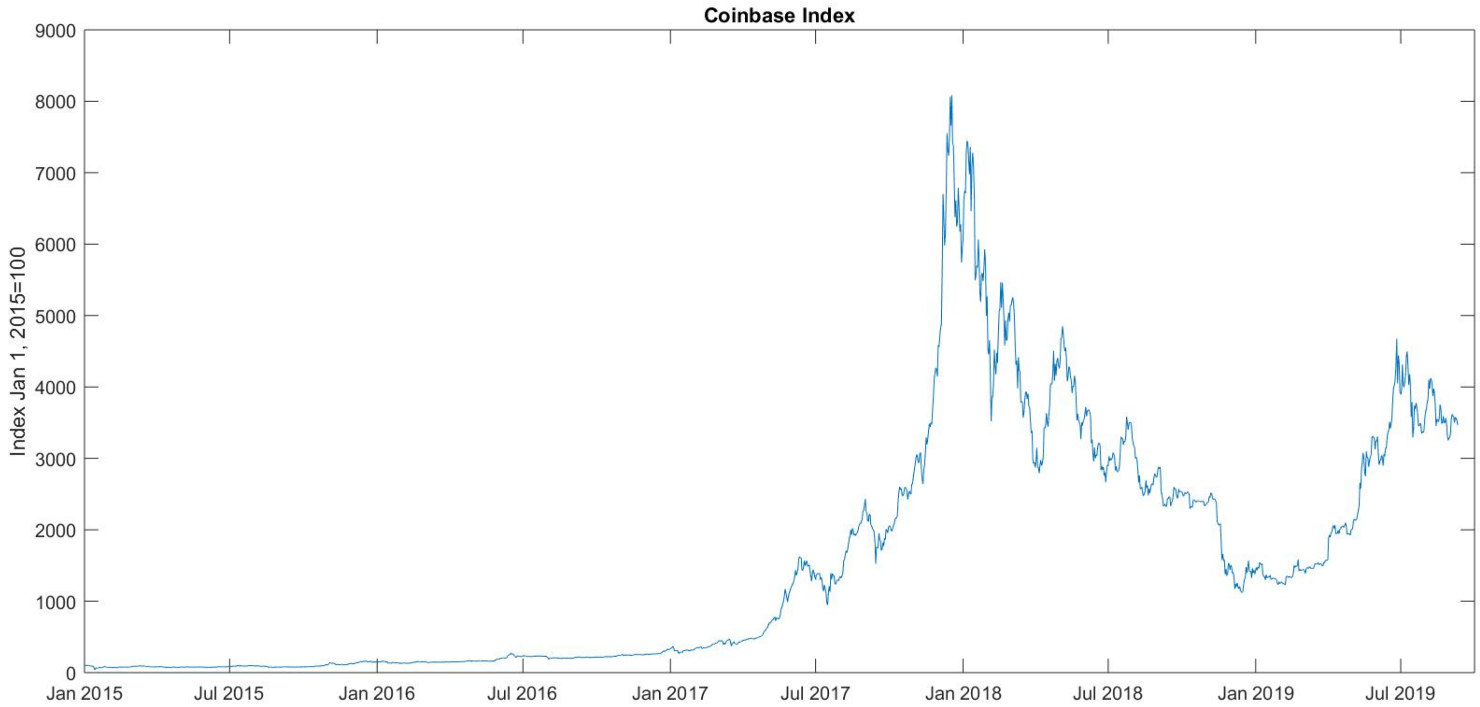

Figure 1: Coinbase Cryptocurrency Index

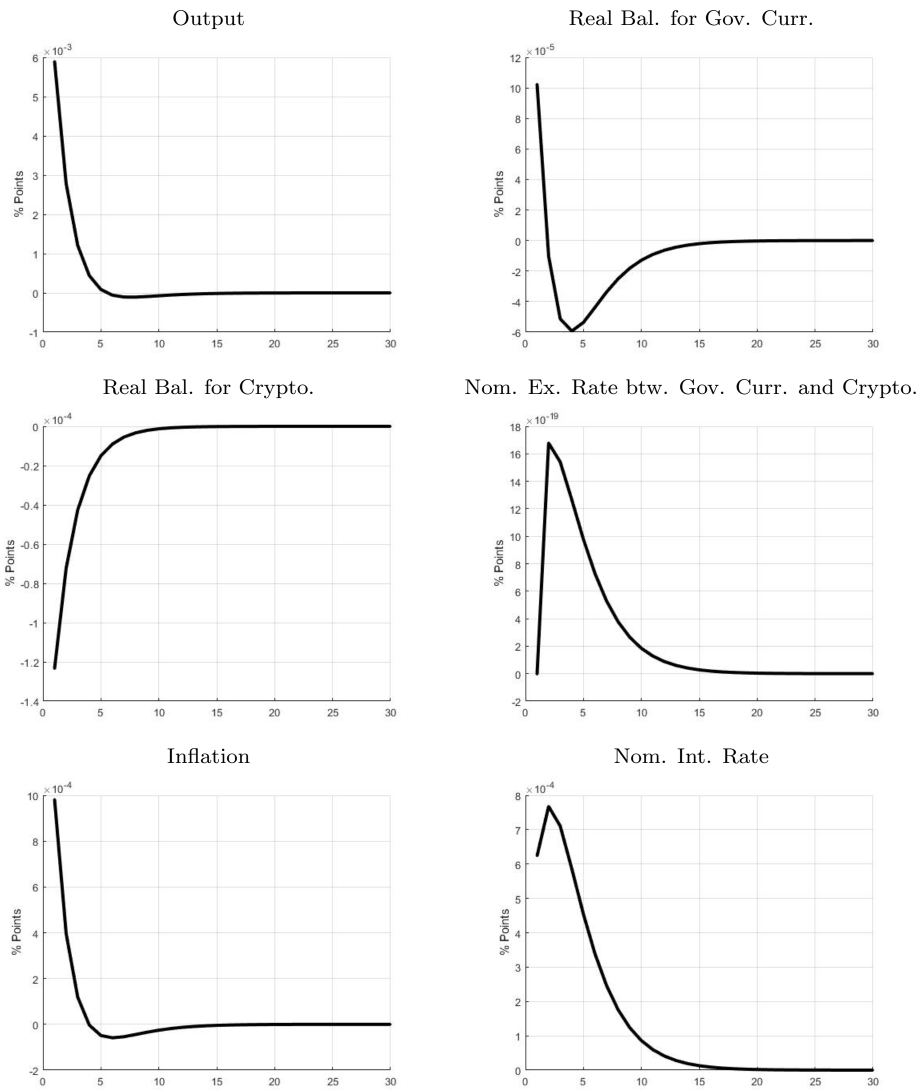

Figure 2: Responses to Preferences Shock

Notes: Simulated 1% shock to household preference.

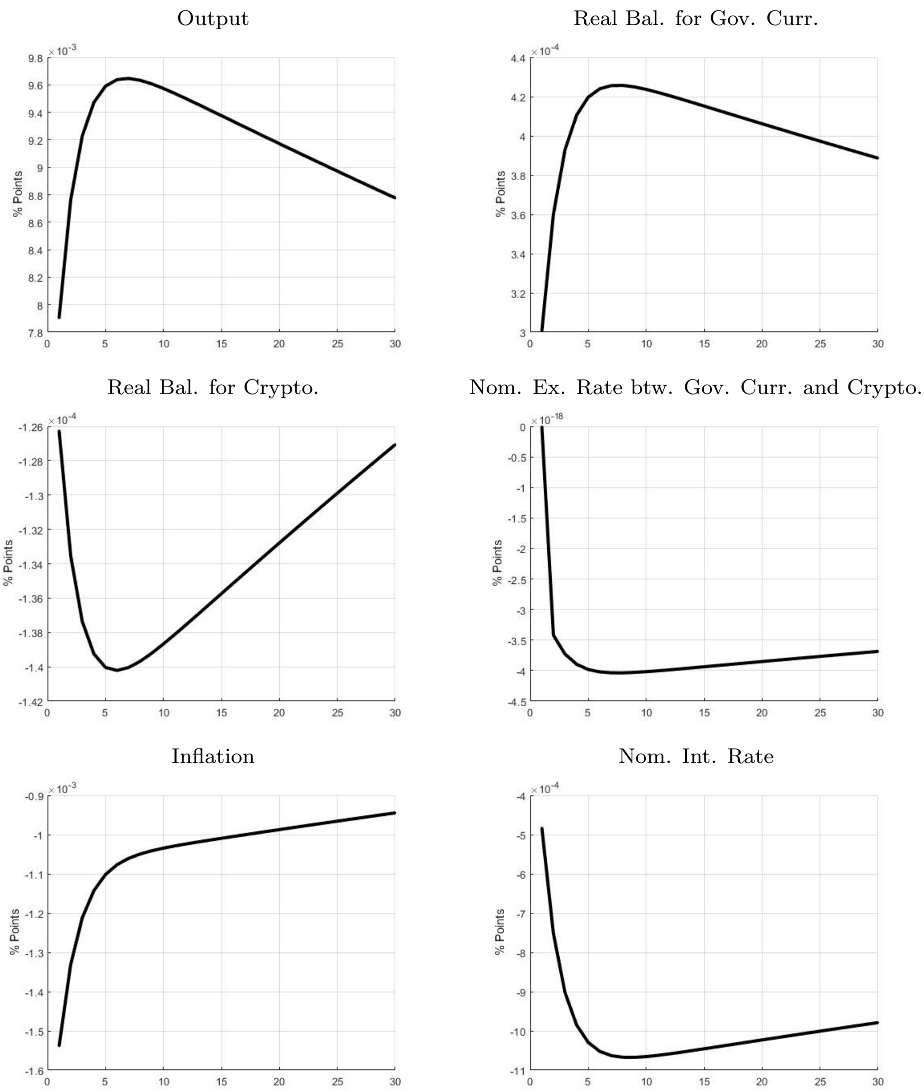

Figure 3: Responses to Technology Shock

Notes: Simulated 1% shock to technology.

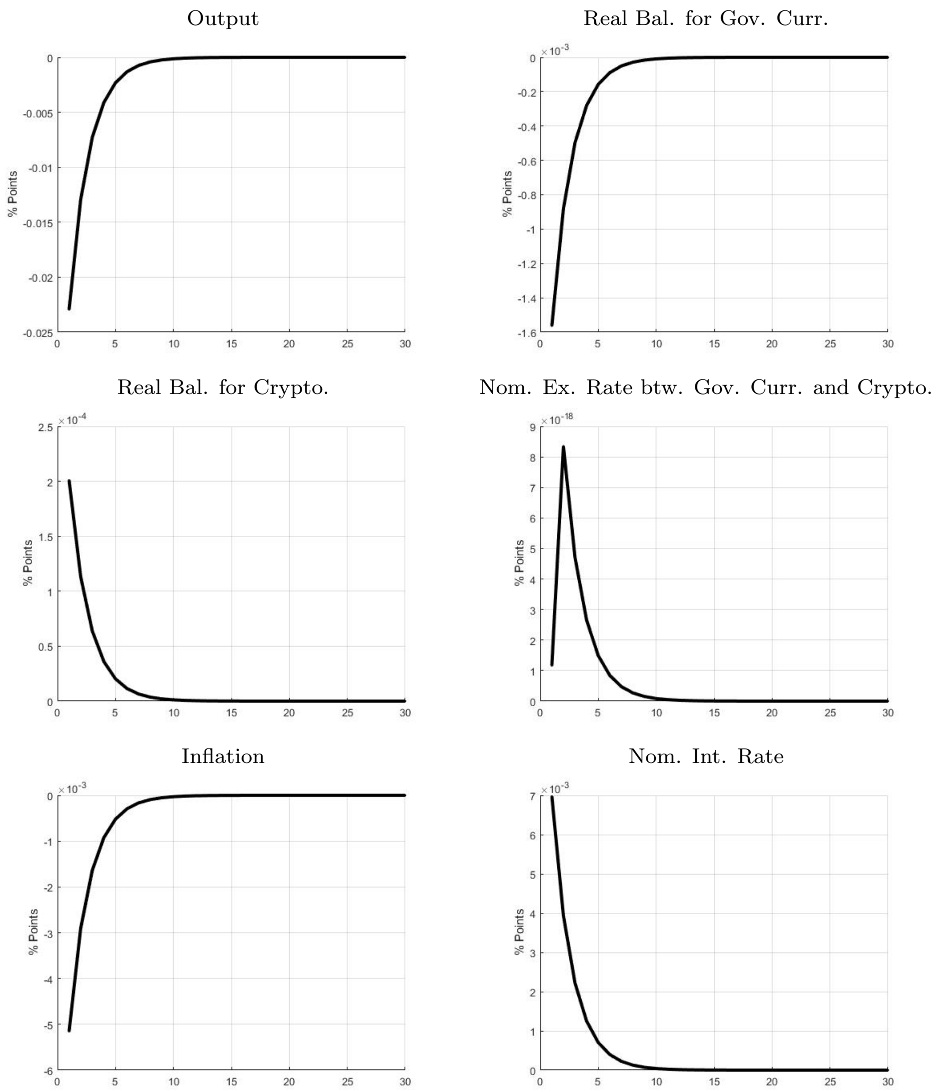

Figure 4: Responses to Monetary Policy Shock

Notes: Simulated 1% shock to monetary policy.

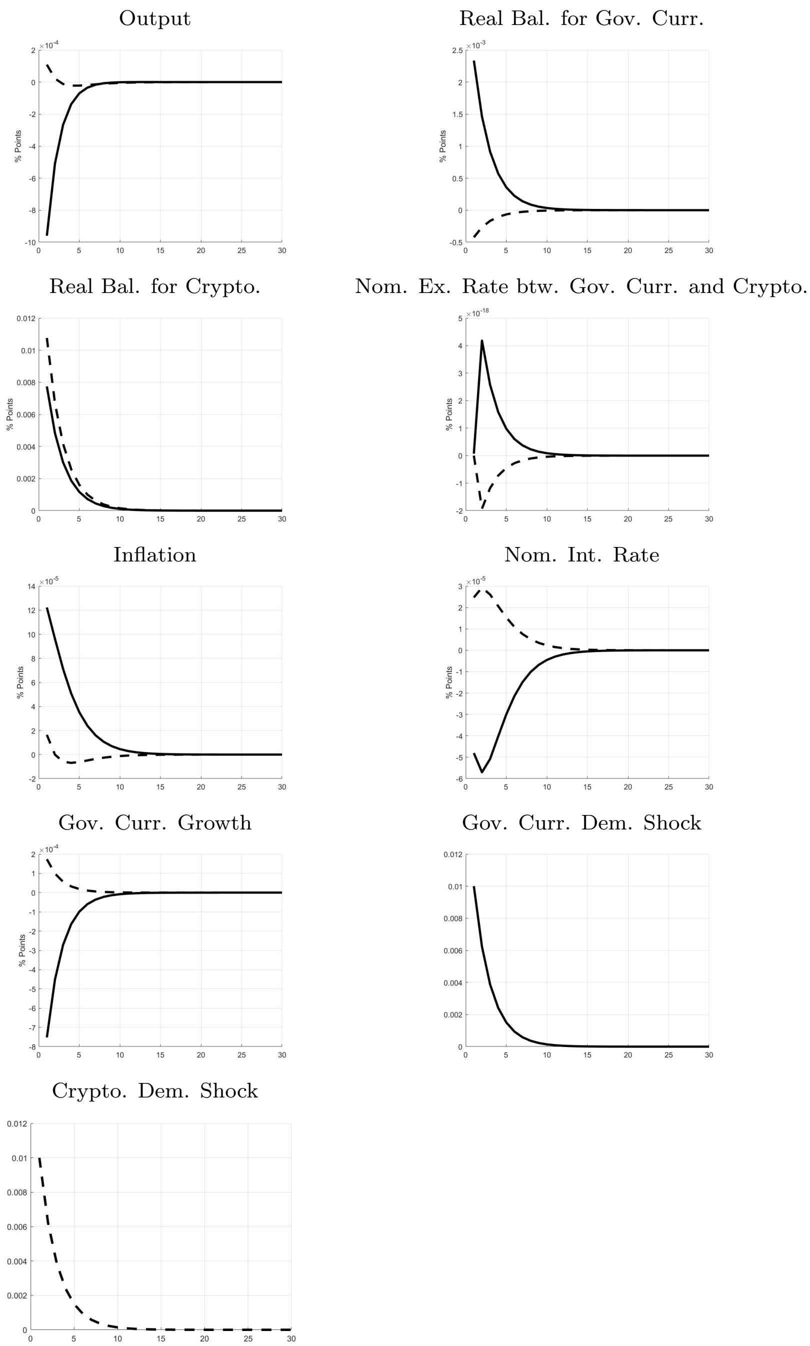

Figure 5: Responses to Government Currency and Cryptocurrency Demand Shocks

Notes: Simulated 1% shocks to government currency and cryptocurrency demands. Solid lines denote the responses to a government currency demand shock, whereas dashed lines represent the responses to a cryptocurrency demand shock.

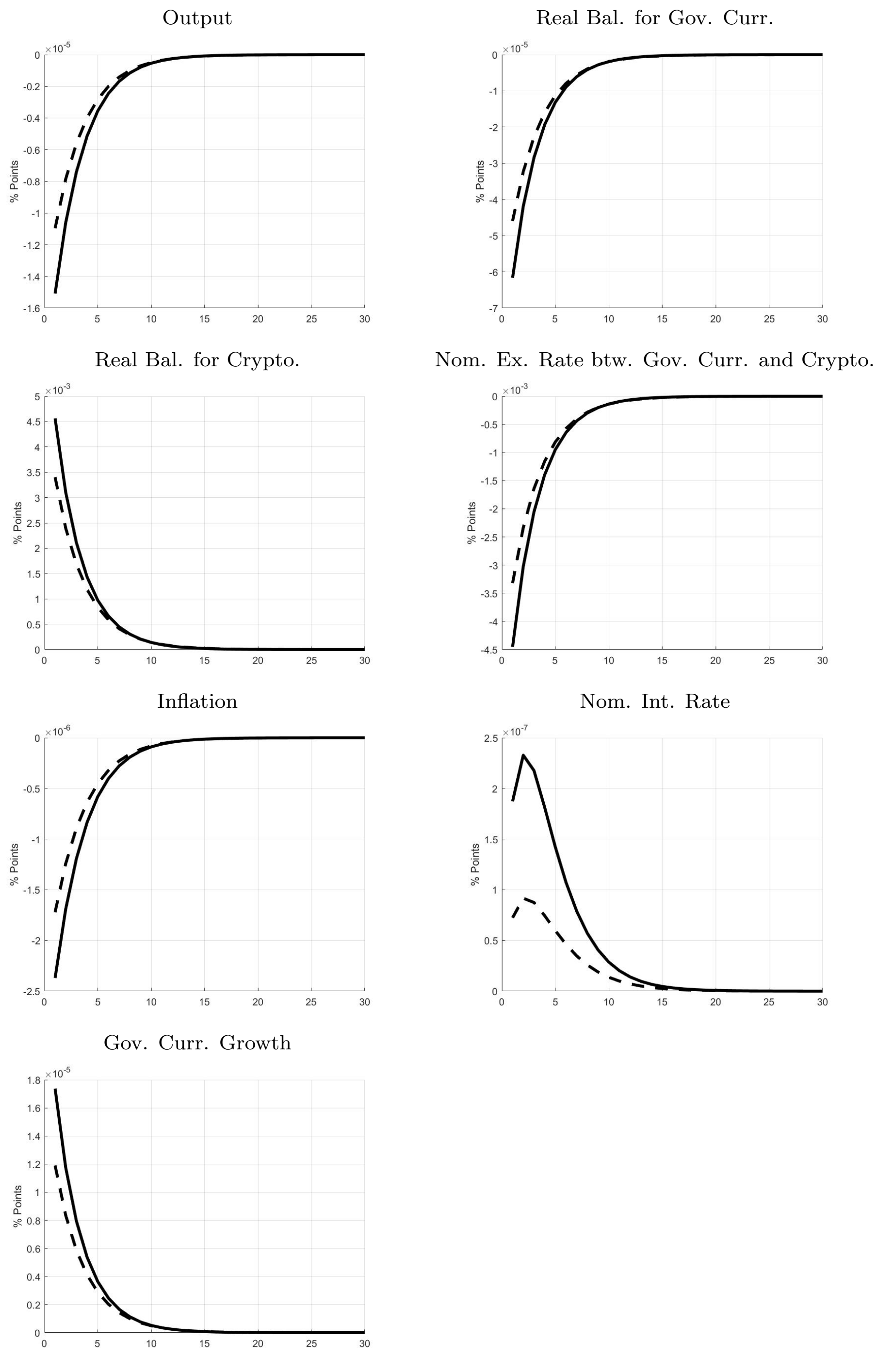

Figure 6: Responses to Cryptocurrency Productivity Shocks

Notes: Simulated 1% shocks to common and specific productivity of cryptocurrency. Solid lines denote the responses to a common productivity shock of cryptocurrency, whereas dashed lines represent the responses to a specific productivity shock of cryptocurrency.

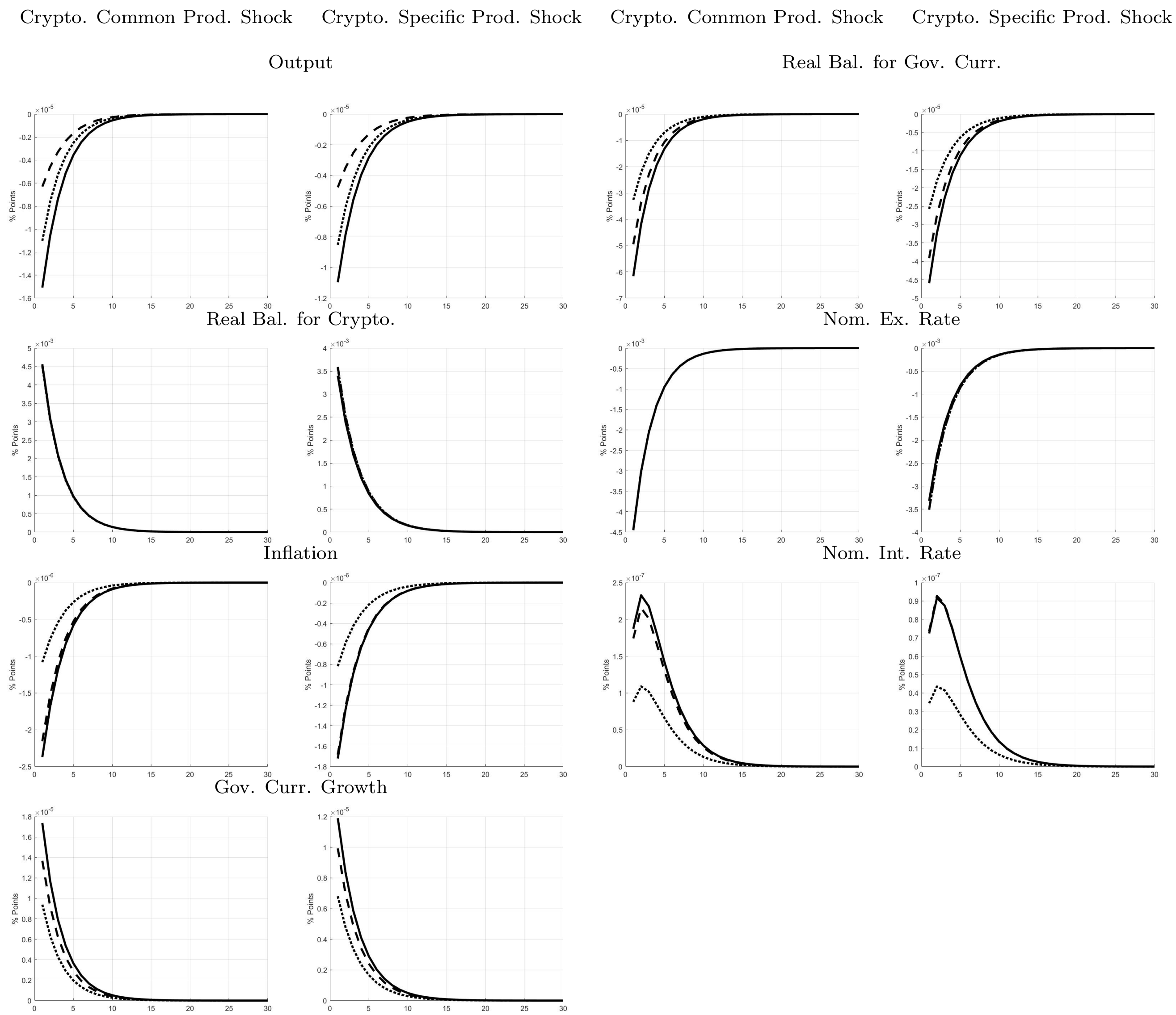

Figure 7: Non-separability vs. Separability: Responses to Cryptocurrency Common and Specific Productivity Shocks

Notes: Simulated 1% shock to common and specific productivity of cryptocurrency. Solid lines denote the IRFs of the benchmark model, whereas the dashed and dotted lines represent the responses of the counterfactual models A (where $\omega_2 = 0$ and $\omega_3 = 0$) and B (where $\omega_3 = 0$), respectively.

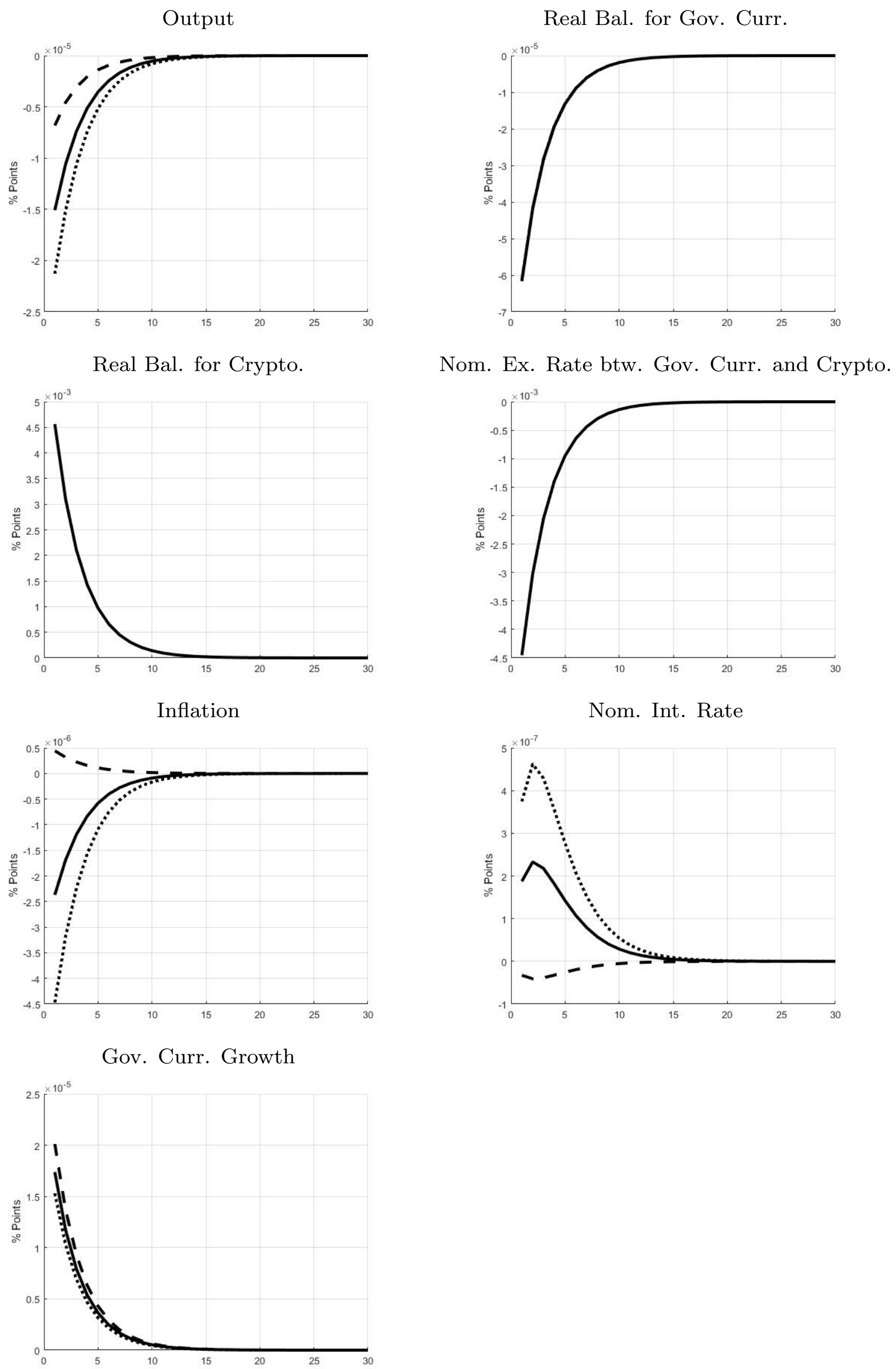

Figure 8: The Role of Monetary Policy: Responses to Cryptocurrency Common Productivity Shock

Notes: Simulated 1% shock to common and specific productivity of cryptocurrency. Solid lines denote the IRFs of the benchmark model, whereas the dashed and dotted lines represent the responses of the model in scenarios 1 and 2, respectively.

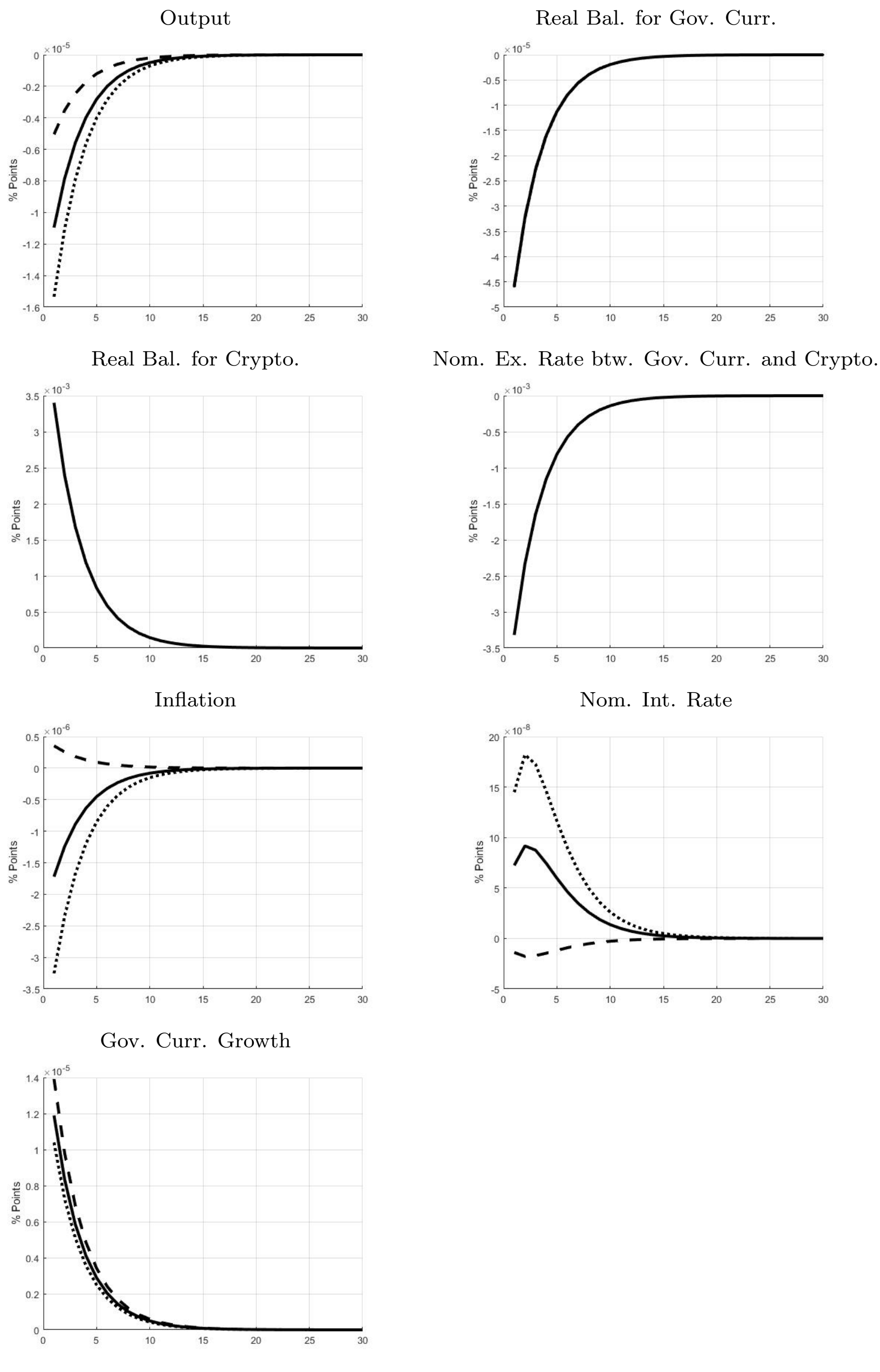

Figure 9: The Role of Monetary Policy: Responses to Cryptocurrency Common Productivity Shock

Notes: Simulated 1% shock to common and specific productivity of cryptocurrency. Solid lines denote the IRFs of the benchmark model, whereas the dashed and dotted lines represent the responses of the model in scenarios 1 and 2, respectively.

Table 1: Exogenous Shocks and Observed Variables

Shocks	**Observed Variables**
Technology Shock	US Industrial Production Index
Shock to Household's Preferences	US Real Private Consumption
Shock to Household's Demand for Government Currency	US Real Balances for Government Currency
Shock to Household's Demand for Cryptocurrency	Real Bitcoin Price
Common Productivity Shock of Cryptocurrency	Real Cumulative Initial Coin Offering (ICO)
Specific Productivity Shock of Cryptocurrency	Real Nvidia Volume Weighted Average Price
Monetary Policy Shock	US Nominal Interest Rate

Notes: The data sources and the construction of all observed variables are reported in Appendix B.

Table 2: Priors and Posteriors for the Endogenous Parameters

Parameter	*Symbol*	*Priors*			*Posteriors*		
		Dist.	Mean	St. Dev.	Mean	Conf.	Inter.
Output El. to Real Bal. of Gov. Currency	ω_2	G	0.200	0.050	0.195	0.102	0.284
Output El. to Real Bal. of Cryptourrency	ω_3	G	0.050	0.010	0.035	0.024	0.046
Income El. of Gov. Currency Demand	γ_1	G	0.015	0.005	0.021	0.009	0.032
Interest Semi-El. of Gov. Currency Demand	γ_2	G	0.150	0.050	0.140	0.066	0.214
El. of Real Bal. of Gov. Curr. wrt Gov. Curr. Dem. Shock	γ_3	G	0.900	0.100	0.664	0.593	0.733
Cross El. of Gov. Cur. Dem. and Crypto. Dem.	γ_4	G	0.500	0.050	0.554	0.467	0.638
Income El. Cryptocurrency Demand	γ_5	G	0.015	0.005	0.013	0.006	0.020
Interest Semi-El. of Cryptocurrency Demand	γ_6	G	0.150	0.050	0.155	0.073	0.236
El. of Real Bal. of Crypto. wrt Crypto. Dem. Shock	γ_7	G	0.800	0.100	1.034	1.014	1.053
Cross El. of Crypto. Dem. and Gov. Cur. Dem.	γ_8	G	0.600	0.100	1.011	0.985	1.037
Ex. Rate Crypto. / Gov. Cur. El. wrt Prod.	ϱ	G	0.900	0.100	0.777	0.638	0.916
Share of Crypto. Common Prod. on Crypto. Tot. Prod.	$\frac{\xi}{\phi}$	G	0.500	0.050	0.572	0.482	0.662
Interest. Rate Smoothing	ρ^r	B	0.800	0.050	0.808	0.765	0.852
Taylor Rule Coef. on Output	ρ^y	B	0.200	0.010	0.153	0.142	0.163
Taylor Rule Coef. on Inflation	ρ^π	G	1.800	0.050	1.980	1.900	2.063
Taylor Rule Coef. on Gov. Currency Growth	ρ^{μ^g}	B	0.200	0.050	0.459	0.368	0.555

Table 3: Priors and Posteriors for the Shock Processes Parameters

Parameter	*Symbol*	*Priors*			*Posteriors*		
		Distr.	Mean	St. Dev.	Mean	Conf.	Inter.
Household's Preference Shock Pers.	ρ^a	B	0.700	0.050	0.668	0.586	0.751
Gov. Cur. Demand Shock Pers.	ρ^{eg}	B	0.650	0.050	0.623	0.548	0.700
Crypto. Demand Shock Pers.	ρ^{ec}	.B	0.550	0.050	0.622	0.554	0.690
Technology Shock Pers.	ρ^z	B	0.900	0.050	0.996	0.992	0.999
Crypto. Common Prod. Shock Pers.	ρ^ξ	B	0.600	0.050	0.679	0.616	0.742
Crypto. Specific Prod. Shock Pers.	ρ^ν	B	0.600	0.050	0.703	0.642	0.765
Household's Preference Shock St. Err.	σ^a	I-G	0.010	Inf	0.278	0.238	0.315
Gov. Cur. Demand Shock St. Err.	σ^{eg}	I-G	0.010	Inf	1.578	0.824	2.320
Crypto. Demand Shock St. Err.	σ^{ec}	I-G	0.010	Inf	3.799	3.065	4.538
Technology Shock St. Err.	σ^z	I-G	0.010	Inf	0.734	0.611	0.853
Crypto. Common Prod. Shock St. Err.	σ^ξ	I-G	0.010	Inf	0.047	0.041	0.054
Crypto. Specific Prod. Shock St. Err.	σ^ν	I-G	0.010	Inf	4.763	4.071	5.436
Monetary Policy Shock St. Err.	σ^r	I-G	0.010	Inf	0.076	0.059	0.091

Table 4: Variance Decomposition (%)

	$\hat{y}_t$	$\hat{\pi}_t$	$\hat{r}_t$	$\hat{m}_t^g$	$\hat{m}_t^c$	$\hat{\chi}_t$
σ^a	3.52	4.04	8.07	0.01	0.00	0.00
σ^{eg}	3.08	2.78	1.41	83.08	7.09	0.00
σ^{ec}	0.24	0.30	2.15	15.85	79.29	0.00
σ^z	89.02	84.20	60.74	0.66	0.00	0.00
σ^ξ	0.00	0.00	0.00	0.00	0.00	0.02
σ^ν	0.00	0.00	0.00	0.32	13.61	99.98
σ^r	4.14	8.68	27.63	0.08	0.00	0.00

Table 5: Forecast Error Variance Decomposition (%)

Period 1						
	$\hat{y}_t$	$\hat{\pi}_t$	$\hat{r}_t$	$\hat{m}_t^g$	$\hat{m}_t^c$	$\hat{\chi}_t$
σ^a	6.41	4.82	6.68	0.00	0.00	0.00
σ^{eg}	5.49	2.41	1.29	83.45	7.17	0.00
σ^{ec}	0.41	0.25	1.97	15.86	80.23	0.00
σ^z	80.49	82.69	28.03	0.30	0.00	0.00
σ^ξ	0.00	0.00	0.00	0.00	0.00	0.02
σ^ν	0.01	0.00	0.00	0.29	12.60	99.98
σ^r	7.19	9.82	62.04	0.09	0.00	0.00

Period 5						
	$\hat{y}_t$	$\hat{\pi}_t$	$\hat{r}_t$	$\hat{m}_t^g$	$\hat{m}_t^c$	$\hat{\chi}_t$
σ^a	1.48	1.84	6.04	0.01	0.00	0.00
σ^{eg}	1.38	1.72	1.02	82.39	7.02	0.00
σ^{ec}	0.08	0.12	1.56	15.76	78.35	0.00
σ^z	95.14	91.69	75.72	1.43	0.00	0.00
σ^ξ	0.00	0.00	0.00	0.00	0.00	0.02
σ^ν	0.00	0.00	0.00	0.34	14.62	99.98
σ^r	1.91	4.63	15.65	0.08	0.00	0.00

Period 12						
	$\hat{y}_t$	$\hat{\pi}_t$	$\hat{r}_t$	$\hat{m}_t^g$	$\hat{m}_t^c$	$\hat{\chi}_t$
σ^a	0.59	0.99	2.54	0.01	0.00	0.00
σ^{eg}	0.55	0.95	0.42	80.39	7.00	0.00
σ^{ec}	0.04	0.07	0.64	15.38	78.11	0.00
σ^z	98.06	95.49	90.49	3.81	0.00	0.00
σ^ξ	0.00	0.00	0.00	0.00	0.00	0.02
σ^ν	0.00	0.00	0.00	0.34	14.89	99.98
σ^r	0.76	2.49	5.92	0.07	0.00	0.00

Period 30						
	$\hat{y}_t$	$\hat{\pi}_t$	$\hat{r}_t$	$\hat{m}_t^g$	$\hat{m}_t^c$	$\hat{\chi}_t$
σ^a	0.25	0.49	1.03	0.01	0.00	0.00
σ^{eg}	0.23	0.46	0.17	76.08	7.00	0.00
σ^{ec}	0.01	0.03	0.26	14.56	78.10	0.00
σ^z	99.19	97.80	96.13	8.97	0.01	0.00
σ^ξ	0.00	0.00	0.00	0.00	0.00	0.02
σ^ν	0.00	0.00	0.00	0.32	14.89	99.98
σ^r	0.32	1.22	2.41	0.07	0.00	0.00

Table 6: Posterior Estimates for the Endogenous Parameters - Benchmark Model vs. Counterfactual Models A and B

Parameter	Symbol	Benchmark			Counterfactual A with $\omega_2 = 0$ and $\omega_3 = 0$			Counterfactual B with $\omega_3 = 0$		
		Mean	Conf. Int.		Mean	Conf. Int.		Mean	Conf. Int.	
Output El. to Real Bal. of Gov. Currency	ω_2	0.195	0.102	0.284	-	-	-	0.241	0.154	0.326
Output El. to Real Bal. of Cryptourrency	ω_3	0.035	0.024	0.046	-	-	-	-	-	-
Income El. of Gov. Currency Demand	γ_1	0.021	0.009	0.032	0.015	0.007	0.022	0.017	0.008	0.025
Interest Semi-El. of Gov. Currency Demand	γ_2	0.140	0.066	0.214	0.151	0.070	0.228	0.137	0.064	0.209
El. of Real Bal. of Gov. Curr. wrt Gov. Curr. Dem. Shock	γ_3	0.664	0.593	0.733	0.948	0.786	1.114	0.716	0.621	0.804
Cross El. of Gov. Cur. Dem. and Crypto. Dem.	γ_4	0.554	0.467	0.638	0.483	0.404	0.561	0.515	0.432	0.597
Income El. Crypto. Demand	γ_5	0.013	0.006	0.020	0.015	0.007	0.023	0.014	0.007	0.022
Interest Semi-El. of Crypto. Demand	γ_6	0.155	0.073	0.236	0.149	0.068	0.226	0.158	0.074	0.241
El. of Real Bal. of Crypto. wrt Crypto. Dem. Shock	γ_7	1.034	1.014	1.053	0.998	0.981	1.015	1.006	0.992	1.019
Cross El. of Crypto. Dem. and Gov. Cur. Dem.	γ_8	1.011	0.985	1.037	1.012	0.969	1.055	1.007	0.975	1.039
Ex. Rate Crypto. / Gov. Cur. El. wrt Prod.	ϱ	0.777	0.638	0.916	0.795	0.651	0.935	0.796	0.654	0.935
Share of Crypto. Common Prod. on Crypto. Tot. Prod.	$\frac{\xi}{\phi}$	0.572	0.482	0.662	0.558	0.471	0.646	0.559	0.471	0.647
Interest. Rate Smoothing	ρ^r	0.808	0.765	0.852	0.801	0.757	0.847	0.804	0.761	0.849
Taylor Rule Coef. on Output	ρ^y	0.153	0.142	0.163	0.153	0.142	0.163	0.153	0.142	0.163
Taylor Rule Coef. on Inflation	ρ^π	1.980	1.898	2.063	1.987	1.904	2.071	1.985	1.902	2.068
Taylor Rule Coef. on Gov. Currency Growth	ρ^{μ^g}	0.459	0.368	0.555	0.448	0.352	0.545	0.456	0.363	0.553